



KEMENTERIAN PENDIDIKAN MALAYSIA

Jabatan Pendidikan Negeri Melaka

#jpnmelakajenamakerajaan1

PROJEK KM²

@ KEMENJADIAN MURID MELAKA

MODUL DLP

KSSM

FASA 2

**MATEMATIK
TAMBAHAN**

TINGKATAN 5

NAMA MURID :

NAMA KELAS :

NAMA GURU :



“PENDIDIKAN BERKUALITI, INSAN TERDIDIK, NEGARA SEJAHTERA”



KEMENTERIAN PENDIDIKAN MALAYSIA
Jabatan Pendidikan Negeri Melaka
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MATA PELAJARAN MATEMATIK TAMBAHAN KSSM TINGKATAN 5**

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EDISI PERTAMA 2021

CETAKAN JABATAN PENDIDIKAN MELAKA

“PENDIDIKAN BERKUALITI, INSAN TERDIDIK, NEGARA SEJAHTERA”

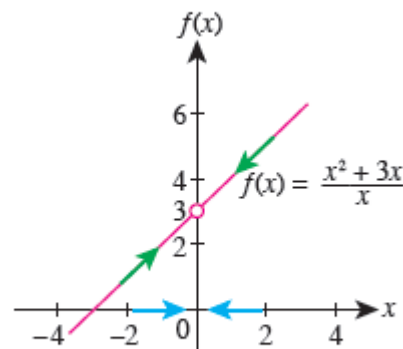
BAB 2 PEMBEZAAN / Chapter 2 : Differentiation

2.1 Had dan Hubungannya dengan Pembezaan

2.1 *Limit and Its Relation to Differentiation*

- ❖ Had merupakan konsep asas dalam operasi pembezaan seperti halaju, v suatu objek pada masa t yang disebut sebagai halaju seketika
- ❖ *The concept of limits has been regarded as a basic concept in differential operations, just like the concept of velocity, v of an object at a certain time t is regarded as its instantaneous velocity at that moment.*

x	$f(x)$
-0.1	2.9
-0.01	2.99
-0.001	2.999
-0.0001	2.9999
\vdots	\vdots
0.0001	3.0001
0.001	3.001
0.01	3.01
0.1	3.1



$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{x^2 + 3x}{x} = 3$$

When x approaches a , where $x \neq a$, the limit for $f(x)$ is L can be written as $\lim_{x \rightarrow a} f(x) = L$.

To find the limit value of a function $f(x)$, we substitute $x = a$ directly into the function $f(x)$. If,

$f(a) \neq \frac{0}{0}$

The value of $\lim_{x \rightarrow a} f(x)$ can be obtained, that is, $\lim_{x \rightarrow a} f(x) = f(a)$.

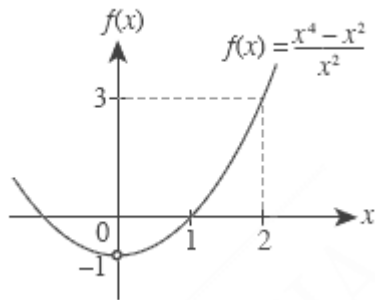
$f(a) = \frac{0}{0}$

Determine $\lim_{x \rightarrow a} f(x)$ by using the following methods:

- Factorisation
- Rationalising the numerator or denominator of the function.

<p>Contoh 1 : Example 1 :</p> $\lim_{x \rightarrow 4} \frac{3 - \sqrt{x}}{x + 2}$ <p>Penyelesaian : Solution :</p> $\begin{aligned} \lim_{x \rightarrow 4} \frac{3 - \sqrt{x}}{x + 2} &= \frac{3 - \sqrt{4}}{4 + 2} \\ &= \frac{3 - 2}{4 + 2} \\ &= \frac{1}{6} \end{aligned}$	<p>Contoh 2 : Example 2 :</p> $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ <p>Penyelesaian : Solution :</p> <p>Apabila $x = 1$, pecahan adalah dalam bentuk tak tentu, $\frac{0}{0}$. Jadi, lakukan operasi pemfaktoran. When $x = 1$, the fraction is in the indeterminate form, $\frac{0}{0}$. Thus, we can use factorization.</p> $\begin{aligned} \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} &= \lim_{x \rightarrow 1} \frac{(x + 1)(x - 1)}{x - 1} \\ &= \lim_{x \rightarrow 1} (x + 1) \\ &= 1 + 1 \\ &= 2 \end{aligned}$
<p>Contoh 3 : Example 3 :</p> $\lim_{x \rightarrow 0} \frac{\sqrt{x + 1} - 1}{x}$ <p>Penyelesaian : Solution :</p> <p>Apabila $x = 0$, pecahan adalah dalam bentuk tak tentu, $\frac{0}{0}$. Jadi, rasionalkan pengangka dengan konjugat. When $x=0$, the fraction is in the indeterminate form $\frac{0}{0}$. Therefore, there is a need to rationalize the numerator by multiplying it with its conjugate.</p> $\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{x + 1} - 1}{x} &= \lim_{x \rightarrow 0} \left(\frac{\sqrt{x + 1} - 1}{x} \right) \left(\frac{\sqrt{x + 1} + 1}{\sqrt{x + 1} + 1} \right) \\ &= \lim_{x \rightarrow 0} \frac{(x + 1) - 1}{x(\sqrt{x + 1} + 1)} \\ &= \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x + 1} + 1)} \end{aligned}$	

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+1} + 1} \\
 &= \frac{1}{\sqrt{0+1} + 1} \\
 &= \frac{1}{1+1} \\
 &= \frac{1}{2}
 \end{aligned}$$



Rajah di sebelah menunjukkan sebagai daripada graf

$f(x) = \frac{x^4 - x^2}{x^2}, x \neq 0$. Berdasarkan graf, cari

The diagram on the right shows a part of the graph $f(x) = \frac{x^4 - x^2}{x^2}, x \neq 0$. Based on the graph, find

- a) $f(0)$ b) $\lim_{x \rightarrow 0} f(x)$ c) $\lim_{x \rightarrow 2} f(x)$

Penyelesaian :

Solution :

- a) There is no value for $x = 0$. Therefore, $f(0)$ cannot be defined at $x = 0$.
 b) When $x \rightarrow 0$ either from the left or from the right, $f(x) \rightarrow -1$. Thus, $\lim_{x \rightarrow 0} f(x) = -1$.
 c) When $x \rightarrow 2$ either from the left or from the right, $f(x) \rightarrow 3$. Thus, $\lim_{x \rightarrow 2} f(x) = 3$.

Latihan Kendiri :

1. Cari had bagi setiap fungsi yang berikut apabila $x \rightarrow 0$.

Find the limit for each of the following functions when $x \rightarrow 0$.

a) $x^2 + x - 3$

b) $\sqrt{x+1}$

[-3]

[1]

c) $\frac{x+4}{x-2}$

d) $\frac{a}{ax+a}$

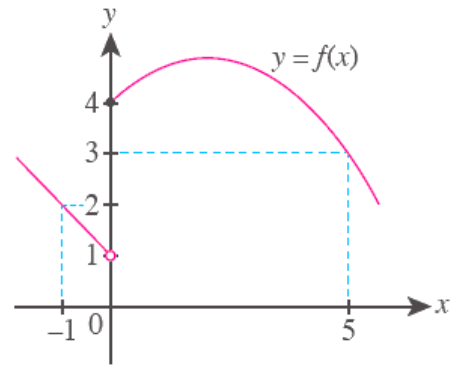
[-2]

[1]

<p>2. Tentukan had bagi setiap fungsi yang berikut. <i>Determine the limit for each of the following functions.</i></p>	
<p>a) $\lim_{x \rightarrow 0} (3x - 1)$</p> <p style="text-align: right;">[-1]</p>	<p>b) $\lim_{x \rightarrow -3} \sqrt{10 - 2x}$</p> <p style="text-align: right;">[4]</p>
<p>c) $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x + 3}$</p> <p style="text-align: right;">[-5]</p>	<p>d) $\lim_{x \rightarrow 6} \frac{x - 6}{x^2 - 36}$</p> <p style="text-align: right;">[$\frac{1}{12}$]</p>
<p>e) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$</p> <p style="text-align: right;">[$\frac{1}{4}$]</p>	<p>f) $\lim_{x \rightarrow 0} \frac{1 - \sqrt{2x + 1}}{2x^2 - x}$</p> <p style="text-align: right;">[1]</p>
<p>g) $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$</p> <p style="text-align: right;">[4]</p>	<p>h) $\lim_{x \rightarrow 3} \frac{3 - \sqrt{2x + 3}}{x - 3}$</p> <p style="text-align: right;">[$-\frac{1}{3}$]</p>
<p>3. Cari nilai bagi setiap had yang berikut. <i>Find the value for each of the following limits.</i></p>	
<p>a) $\lim_{x \rightarrow 0} \frac{x^2 - 2}{x^3 - 4x}$</p> <p style="text-align: right;">[$\frac{1}{2}$]</p>	<p>b) $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{2x^2 - 5x - 3}$</p> <p style="text-align: right;">[$\frac{2}{7}$]</p>

<p>c) $\lim_{x \rightarrow 3} \frac{x^3 - 5x^2 + 6x}{x^2 - 3x}$</p> <p style="text-align: right;">[1]</p>	<p>d) $\lim_{x \rightarrow 0} \frac{5x}{3 - \sqrt{x+9}}$</p> <p style="text-align: right;">[-30]</p>
<p>e) $\lim_{x \rightarrow 4} \frac{x-4}{2 - \sqrt{8-x}}$</p> <p style="text-align: right;">[4]</p>	<p>f) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7}$</p> <p style="text-align: right;">[$\frac{1}{6}$]</p>

4. Rajah di sebelah menunjukkan sebahagian daripada graf fungsi $y = f(x)$.
The diagram on the right shows a part of the function graph $y = f(x)$.



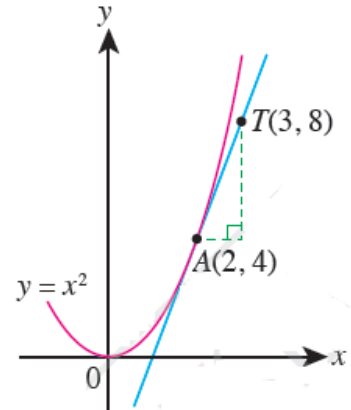
- a) Berdasarkan graf,
Based on the graph,
- (i) cari $f(0)$,
find $f(0)$,
 - (ii) tentukan sama ada $\lim_{x \rightarrow 0} f(x)$ wujud atau tidak. Jelaskan.
Determine whether $\lim_{x \rightarrow 0} f(x)$ exists or not.
Explain.
- b) Seterusnya, cari
Then, find
- (i) $\lim_{x \rightarrow -1} f(x)$
 - (ii) $\lim_{x \rightarrow 5} f(x)$

2.1.2 Terbitan Pertama suatu fungsi $f(x)$ melalui pembezaan dengan Prinsip Pertama

2.1.2 *First Derivative of a function $f(x)$ by using first principles*

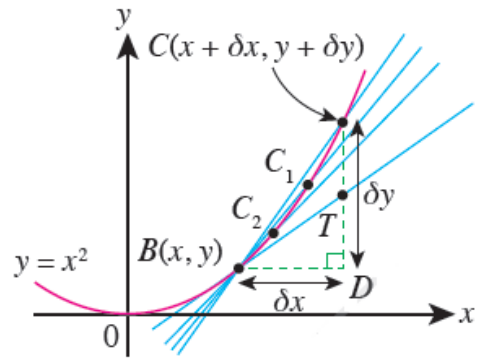
Tangen kepada suatu lengkung di suatu titik ialah satu garis lurus yang menyentuh lengkung pada titik itu. Dalam rajah di sebelah, garis lurus AT dengan koordinat A dan T masing-masing ialah $(2, 4)$ dan $(3, 8)$ ialah tangen kepada lengkung $y = x^2$ di titik A .

A tangent to a curve at a point is a straight line that touches the curve at only that point. In the diagram on the right, straight line AT is a tangent to the curve $y = x^2$ at the point A with the coordinates of A and T being $(2, 4)$ and $(3, 8)$ respectively.



$$\text{Gradient of tangent } AT = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{3 - 2} = 4$$

$$\begin{aligned} \text{Gradient of the line } BC &= \frac{CD}{BD} \\ &= \frac{(y + \delta y) - y}{(x + \delta x) - x} \\ &= \frac{\delta y}{\delta x} \end{aligned}$$



$$\begin{aligned} \text{Gradient of the curve at } B &= \text{Gradient of tangent } BT \\ &= \text{Value of } \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} \end{aligned}$$

$$\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

$$y = f(x) = x^2$$

Penyelesaian :
Solution :

$$\begin{aligned} \delta y &= f(x + \delta x) - f(x) \\ &= (x + \delta x)^2 - x^2 \\ &= x^2 + 2x(\delta x) + (\delta x)^2 - x^2 \\ &= 2x(\delta x) + (\delta x)^2 \end{aligned}$$

$$\begin{aligned} \frac{\delta y}{\delta x} &= \frac{2x(\delta x) + (\delta x)^2}{\delta x} \\ &= 2x + \delta x \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} \\ &= \lim_{\delta x \rightarrow 0} (2x + \delta x) \\ &= 2x + 0 \end{aligned}$$

$$\frac{dy}{dx} = 2x$$

Terbitan pertama bagi suatu fungsi $y = f(x)$ dengan menggunakan idea $\lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$ seperti ini disebut sebagai **PEMBEZAAN DENGAN PRINSIP PERTAMA**.

Contoh 1 :

Example 1 :

$$y = 3x$$

Penyelesaian :

Solution :

$$\begin{aligned}\delta y &= f(x + \delta x) - f(x) \\ &= 3(x + \delta x) - 3x \\ &= 3x + 3\delta x - 3x \\ &= 3\delta x\end{aligned}$$

$$\frac{\delta y}{\delta x} = 3$$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} \\ &= \lim_{\delta x \rightarrow 0} 3\end{aligned}$$

$$\frac{dy}{dx} = 3$$

Contoh 2 :

Example 2 :

$$y = 3x^2$$

Penyelesaian :

Solution :

$$\begin{aligned}\delta y &= f(x + \delta x) - f(x) \\ &= 3(x + \delta x)^2 - 3x^2 \\ &= 3[x^2 + 2x(\delta x) + (\delta x)^2] - 3x^2 \\ &= 3x^2 + 6x(\delta x) + 3(\delta x)^2 - 3x^2 \\ &= 6x(\delta x) + 3(\delta x)^2\end{aligned}$$

$$\frac{\delta y}{\delta x} = 6x + 3\delta x$$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} \\ &= \lim_{\delta x \rightarrow 0} (6x + 3\delta x) \\ &= 6x + 3(0)\end{aligned}$$

$$\frac{dy}{dx} = 6x$$

Contoh 3 :

Example 3 :

$$y = 3x^3$$

Penyelesaian :

Solution :

$$\begin{aligned}\delta y &= f(x + \delta x) - f(x) \\ &= 3(x + \delta x)^3 - 3x^3 \\ &= 3(x + \delta x)(x + \delta x)^2 - 3x^3 \\ &= 3(x + \delta x)[x^2 + 2x(\delta x) + (\delta x)^2] - 3x^3 \\ &= 3[x^3 + 2x^2(\delta x) + x(\delta x)^2 + x^2(\delta x) + 2x(\delta x)^2 + (\delta x)^3] - 3x^3 \\ &= 3[x^3 + 3x^2(\delta x) + 3x(\delta x)^2 + (\delta x)^3] - 3x^3 \\ &= 3x^3 + 9x^2(\delta x) + 9x(\delta x)^2 + 3(\delta x)^3 - 3x^3 \\ &= 9x^2(\delta x) + 9x(\delta x)^2 + 3(\delta x)^3\end{aligned}$$

$$\frac{\delta y}{\delta x} = 9x^2 + 9x(\delta x) + 3(\delta x)^2$$

<p><i>Then,</i></p> $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$ $= \lim_{\delta x \rightarrow 0} [9x^2 + 9x(\delta x) + 3(\delta x)^2]$ $= 9x^2 + 9x(0) + 3(0)^2$ $\frac{dy}{dx} = 9x^2$	
Latihan Kendiri / Self Exercise	
<p>1. Cari $\frac{dy}{dx}$ dengan menggunakan prinsip pertama bagi setiap fungsi $y = f(x)$ yang berikut. Find $\frac{dy}{dx}$ by using first principles for each of the following functions $y = f(x)$.</p>	
<p>a) $y = x$</p> <p style="text-align: right;">[1]</p>	<p>b) $y = 5x$</p> <p style="text-align: right;">[5]</p>
<p>c) $y = -4x$</p> <p style="text-align: right;">[-4]</p>	<p>d) $y = 6x^2$</p> <p style="text-align: right;">[12x]</p>
<p>e) $y = -x^2$</p> <p style="text-align: right;">[-2x]</p>	<p>f) $y = 2x^3$</p> <p style="text-align: right;">[6x²]</p>
<p>g) $y = \frac{1}{2}x^2$</p> <p style="text-align: right;">[x]</p>	<p>h) $y = \frac{1}{x}$</p> <p style="text-align: right;">[-$\frac{1}{x^2}$]</p>

<p>2. Diberi $y = 2x^2 - x + 7$, cari $\frac{dy}{dx}$ dengan menggunakan prinsip pertama. <i>Given $y = 2x^2 - x + 7$, find $\frac{dy}{dx}$ by using first principles.</i></p> <p style="text-align: right;">[4x - 1]</p>	<p>3. Dengan menggunakan prinsip pertama, cari fungsi kecerunan bagi lengkung $y = 3 + x - x^2$. <i>By using first principles, find the gradient function to the curve $y = 3 + x - x^2$.</i></p> <p style="text-align: right;">[1 - 2x]</p>
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Latihan Formatif / *Formative Exercise*

<p>1. Diberi bahawa $\lim_{x \rightarrow 2} \frac{x^2 - k}{3x - 6} = \frac{4}{3}$, cari nilai k. <i>Given that $\lim_{x \rightarrow 2} \frac{x^2 - k}{3x - 6} = \frac{4}{3}$, find the value of k.</i></p> <p style="text-align: right;">[k = 4]</p>	<p>2. Jika $\lim_{x \rightarrow -1} \frac{x^2 - 2x - h}{kx + 2} = -2$, cari nilai $h + k$ <i>If $\lim_{x \rightarrow -1} \frac{x^2 - 2x - h}{kx + 2} = -2$, find the value of $h + k$</i></p> <p style="text-align: right;">[4]</p>
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<p>3. Bezakan fungsi berikut terhadap x dengan menggunakan prinsip pertama. <i>Differentiate the following functions with respect to x by using first principles.</i></p>	
<p>a) $y = 5x - 8$</p> <p style="text-align: right;">[5]</p>	<p>b) $y = x^2 - x$</p> <p style="text-align: right;">[2x-1]</p>

<p>c) $y = (x + 1)^2$</p> <p style="text-align: right;">[$2x + 2$]</p>	<p>d) $\frac{1}{4x}$</p> <p style="text-align: right;">[$-\frac{1}{4x^2}$]</p>
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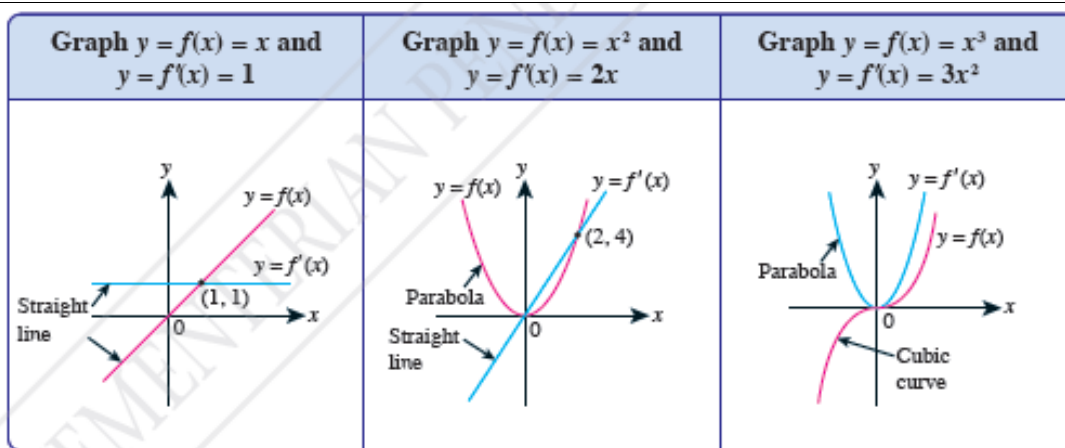
4. Sesaran, s m, bagi seekor tupai yang berlari pada kabel lurus selepas t saat diberi oleh $s(t) = t^2 - 3t$, dengan keadaan $t \geq 0$. Menggunakan prinsip pertama, cari halaju tupai itu apabila $t = 5$.
The displacement of a squirrel running on a straight cable for t seconds is given by $s(t) = t^2 - 3t$, where $t \geq 0$. By using first principles, find the velocity of the squirrel when $t=5$.

[7 ms^{-1}]

2.2 Pembezaan Peringkat Pertama
2.2 The First Derivative

Function	$\frac{dy}{dx}$	Pattern
$y = 3x$	3	$3(1x^{1-1})$
$y = 3x^2$	$6x$	$3(2x^{2-1})$
$y = 3x^3$	$9x^2$	$3(3x^{3-1})$

If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$ or $\frac{d}{dx}(ax^n) = anx^{n-1}$



Contoh 1 :

Example 1 :

Bezakan setiap yang berikut terhadap x .

Differentiate each of the following with respect to x .

a) $-\frac{2}{3}x^6$

Penyelesaian :

Solution :

$$\begin{aligned} \frac{d}{dx} \left(-\frac{2}{3}x^6 \right) &= -\frac{2}{3}(6x^{6-1}) \\ &= -\frac{2}{3}(6x^5) \end{aligned}$$

$$\frac{d}{dx} \left(-\frac{2}{3}x^6 \right) = -4x^5$$

b) $y = \frac{1}{5}\sqrt{x}$

Penyelesaian :

Solution :

$$\begin{aligned} y &= \frac{1}{5}\sqrt{x} \\ &= \frac{1}{5}x^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{5} \left(\frac{1}{2}x^{\frac{1}{2}-1} \right) \\ &= \frac{1}{10}x^{-\frac{1}{2}} \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{10\sqrt{x}}$$

c) $f(x) = \frac{3}{8x^2}$

Penyelesaian :

Solution :

$$\begin{aligned} f(x) &= \frac{3}{8x^2} \\ &= \frac{3}{8}x^{-2} \end{aligned}$$

$$\begin{aligned} f'(x) &= \frac{3}{8}(-2x^{-2-1}) \\ &= -\frac{3}{4}x^{-3} \end{aligned}$$

$$f'(x) = -\frac{3}{4x^3}$$

Contoh 2 :

Example 2 :

a) Jika $f(x) = \frac{3}{4}x^4$, cari $f'(-1)$ dan $f'(\frac{1}{3})$

If $f(x) = \frac{3}{4}x^4$, find $f'(-1)$ and $f'(\frac{1}{3})$

b) Diberi bahawa $y = 9\sqrt[3]{x}$, cari nilai $\frac{dy}{dx}$ apabila $x = 8$.

Given that $y = 9\sqrt[3]{x}$, find the value of $\frac{dy}{dx}$ when $x = 8$.

Penyelesaian :

Solution :

(a) $f(x) = \frac{3}{4}x^4$
 $f'(x) = \frac{3}{4}(4x^{4-1})$
 $= 3x^3$
 $f'(-1) = 3(-1)^3$
 $= -3$
 $f'\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3$
 $= \frac{1}{9}$

(b) $y = 9\sqrt[3]{x}$
 $= 9x^{\frac{1}{3}}$
 $\frac{dy}{dx} = 9\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)$
 $= 3x^{-\frac{2}{3}}$
 When $x = 8$, $\frac{dy}{dx} = 3(8)^{-\frac{2}{3}}$
 $= \frac{3}{4}$

Jika $f(x)$ dan $g(x)$ ialah suatu fungsi, maka
 If $f(x)$ and $g(x)$ are functions, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

Contoh :
 Example :

Bezakan setiap yang berikut terhadap x .
 Differentiate each of the following with respect to x .

(a) $5x^3 + \frac{3}{4}x^4$

(b) $x(\sqrt{x} - 9)$

(c) $\frac{(2x + 1)(x - 1)}{x}$

Penyelesaian :
 Solution :

(a) $\frac{d}{dx}\left(5x^3 + \frac{3}{4}x^4\right) = \frac{d}{dx}(5x^3) + \frac{d}{dx}\left(\frac{3}{4}x^4\right)$
 $= 5(3x^{3-1}) + \frac{3}{4}(4x^{4-1})$
 $\frac{d}{dx}\left(5x^3 + \frac{3}{4}x^4\right) = 15x^2 + 3x^3$

(b) Katakan $f(x) = x(\sqrt{x} - 9)$
 $= x^{\frac{3}{2}} - 9x$
 $f'(x) = \frac{3}{2}x^{\frac{3}{2}-1} - 9(1x^{1-1})$
 $= \frac{3}{2}x^{\frac{1}{2}} - 9$
 $f'(x) = \frac{3}{2}\sqrt{x} - 9$

$$\begin{aligned}
 \text{(c) Katakan } y &= \frac{(2x + 1)(x - 1)}{x} \\
 &= \frac{2x^2 - x - 1}{x} \\
 &= 2x - 1 - x^{-1} \\
 \frac{dy}{dx} &= \frac{d}{dx}(2x) - \frac{d}{dx}(1) - \frac{d}{dx}(x^{-1}) \\
 &= 2x^{1-1} - 0x^{0-1} - (-1x^{-1-1}) \\
 &= 2 + x^{-2} \\
 \frac{dy}{dx} &= 2 + \frac{1}{x^2}
 \end{aligned}$$

Latihan Kendiri / *Self Exercise*

1. Cari terbitan pertama bagi setiap fungsi yang berikut terhadap x .
Find the first derivative for each of the following functions with respect to x .

a) $\frac{4}{5}x^{10}$

b) $-2x^4$

[$8x^9$]

[$-8x^3$]

c) $\frac{3}{4x^8}$

d) $\frac{6}{\sqrt[3]{x}}$

[$-\frac{6}{x^9}$]

[$-\frac{2}{\sqrt[3]{x^4}}$]

2. Bezakan setiap fungsi yang berikut terhadap x .
Differentiate each of the following functions with respect to x .

a) $4x^2 + 6x - 1$

$[8x + 6]$

b) $\frac{4}{5}\sqrt{x} + \frac{2}{\sqrt{x}}$

$[\frac{2}{5\sqrt{x}} - \frac{1}{\sqrt{x^3}}]$

c) $(9 - 4x)^2$

$[32x - 72]$

d) $y = 4x^2(5 - \sqrt{x})$

$[40x - 10\sqrt{x^3}]$

e) $y = \left(x^2 + \frac{4}{x}\right)^2$

$[4x^3 + 8 - \frac{32}{x^3}]$

f) $y = \frac{(4x-1)(1-x)}{\sqrt{x}}$

$[\frac{5}{2\sqrt{x}} - 6\sqrt{x} + \frac{1}{2\sqrt{x^3}}]$

3) Cari nilai $\frac{dy}{dx}$ pada setiap nilai x yang diberi.

Find the value of $\frac{dy}{dx}$ for each of the given value of x .

(a) $y = x^2 - 2x, x = \frac{1}{2}$

(b) $y = \sqrt{x}(2 - x), x = 9$

(c) $y = \frac{x^2 + 4}{x^2}, x = 2$

- a) -1
b) $-4\frac{1}{6}$
c) -1

Terbitan Pertama Fungsi Gubahan
First Derivative of Composite Function

Petua Rantai :
Chain Rule :

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = g(u)$ and $u = h(x)$, then differentiating y with respect to x will give

$$f'(x) = g'(u) \times h'(x)$$

That is,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Contoh :

Example :

(a) $y = (3x^2 - 4x)^7$

(b) $y = \frac{1}{(2x + 3)^3}$

(c) $y = \sqrt{6x^2 + 8}$

Penyelesaian :

Solution :

(a) Let $u = 3x^2 - 4x$ and $y = u^7$
Then, $\frac{du}{dx} = 6x - 4$ and $\frac{dy}{du} = 7u^6$

With chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= 7u^6(6x - 4) \\ &= 7(3x^2 - 4x)^6(6x - 4) \\ &= (42x - 28)(3x^2 - 4x)^6 \\ \frac{dy}{dx} &= 14(3x - 2)(3x^2 - 4x)^6 \end{aligned}$$

(b) Let $u = 2x + 3$ and $y = \frac{1}{u^3} = u^{-3}$
Then, $\frac{du}{dx} = 2$ and $\frac{dy}{du} = -3u^{-3-1} = -\frac{3}{u^4}$

With chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= -\frac{3}{u^4}(2) \\ \frac{dy}{dx} &= -\frac{6}{(2x + 3)^4} \end{aligned}$$

(c) Let $u = 6x^2 + 8$ and $y = \sqrt{u} = u^{\frac{1}{2}}$
Then, $\frac{du}{dx} = 12x$ and $\frac{dy}{du} = \frac{1}{2}u^{\frac{1}{2}-1} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$

With chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= \frac{1}{2\sqrt{u}}(12x) \\ &= \frac{12x}{2\sqrt{6x^2 + 8}} \\ \frac{dy}{dx} &= \frac{6x}{\sqrt{6x^2 + 8}} \end{aligned}$$

Latihan Kendiri / Self Exercise :

Bezakan setiap ungkapan berikut terhadap x .

Differentiate each of the following expressions with respect to x .

1. $(x + 4)^5$

2. $(2x - 3)^4$

$[5(x + 4)^4]$

$[8(2x - 3)^3]$

<p>3. $\frac{1}{3}(x + 4)^5$</p> <p style="text-align: right;">$[(x + 4)^4]$</p>	<p>4. $(4x^2 - 5)^7$</p> <p style="text-align: right;">$[[56x(4x^2 - 5)^6]$</p>
<p>5. $(\frac{1}{6}x + 2)^8$</p> <p style="text-align: right;">$[\frac{4}{3}(\frac{1}{6}x + 2)^7]$</p>	<p>6. $\frac{2}{3}(5 - 2x)^9$</p> <p style="text-align: right;">$[-12(5 - 2x)^8]$</p>
<p>7. $(1 - x - x^2)^3$</p> <p style="text-align: right;">$[-3(2x + 1)(1 - x - x^2)^2]$</p>	<p>8. $(2x^3 - 4x + 1)^{-10}$</p> <p style="text-align: right;">$[-\frac{20(3x^2 - 2)}{(2x^3 - 4x + 1)^{11}]$</p>
<p>9. $\frac{1}{3x+2}$</p> <p style="text-align: right;">$[-\frac{3}{(3x+2)^2}]$</p>	<p>10. $\frac{1}{(2x-7)^3}$</p> <p style="text-align: right;">$[-\frac{6}{(2x-7)^4}]$</p>
<p>11. $\frac{5}{(3-4x)^5}$</p> <p style="text-align: right;">$[\frac{100}{(3-4x)^6}]$</p>	<p>12. $\frac{3}{4(5x-6)^8}$</p> <p style="text-align: right;">$[-\frac{30}{(5x-6)^9}]$</p>

<p>13. $\sqrt{2x - 7}$</p> <p style="text-align: right;">$\left[\frac{1}{\sqrt{2x-7}}\right]$</p>	<p>14. $\sqrt{6 - 3x}$</p> <p style="text-align: right;">$\left[-\frac{3}{2\sqrt{6-3x}}\right]$</p>	
<p>15. $\sqrt{3x^2 + 5}$</p> <p style="text-align: right;">$\left[\frac{3x}{\sqrt{3x^2+5}}\right]$</p>	<p>16. $\sqrt{x^2 - x + 1}$</p> <p style="text-align: right;">$\left[\frac{2x-1}{2\sqrt{x^2-x+1}}\right]$</p>	
<p>Cari nilai bagi $\frac{dy}{dx}$ pada setiap nilai x atau nilai y yang diberi berikut. Find the value of $\frac{dy}{dx}$ for each of the given value of x and y.</p>		
<p>1. $y = (2x + 5)^4, x = 1$</p> <p style="text-align: right;">[2744]</p>	<p>2. $y = \sqrt{5 - 2x}, x = \frac{1}{2}$</p> <p style="text-align: right;">$\left[-\frac{1}{2}\right]$</p>	<p>3. $y = \frac{1}{2x-3}, y = 1$</p> <p style="text-align: right;">[-2]</p>

Terbitan Pertama bagi Suatu Fungsi yang melibatkan **HASIL DARAB** dan **BAHAGI** Ungkapan Algebra

First Derivative of a function involving product and quotient of algebra expressions

The Product Rule

If u and v are functions of x , then

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

The Quotient Rule

If u and v are functions of x , and $v(x) \neq 0$, then

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Contoh :

Example :

1. Bezakan setiap yang berikut terhadap x .

Differentiate each of the following with respect to x .

a) $(x^2 + 1)(x - 3)^4$

Penyelesaian :

Solution :

(a) Given $y = (x^2 + 1)(x - 3)^4$.

Let $u = x^2 + 1$

and $v = (x - 3)^4$

We get $\frac{du}{dx} = 2x$

and $\frac{dv}{dx} = 4(x - 3)^{4-1} \cdot \frac{d}{dx}(x - 3)$
 $= 4(x - 3)^3$

Hence, $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
 $= (x^2 + 1) \times 4(x - 3)^3 + (x - 3)^4 \times 2x$
 $= 4(x^2 + 1)(x - 3)^3 + 2x(x - 3)^4$
 $= 2(x - 3)^3 [2(x^2 + 1) + x(x - 3)]$
 $\frac{dy}{dx} = 2(x - 3)^3 (3x^2 - 3x + 2)$

b) $(3x + 2)\sqrt{4x - 1}$

Penyelesaian :

Solution :

(b) Given $y = (3x + 2)\sqrt{4x - 1}$.

Let $u = 3x + 2$

and $v = \sqrt{4x - 1} = (4x - 1)^{\frac{1}{2}}$

We get $\frac{du}{dx} = 3$

and $\frac{dv}{dx} = \frac{1}{2}(4x - 1)^{\frac{1}{2}-1} \cdot \frac{d}{dx}(4x - 1)$
 $= \frac{1}{2}(4x - 1)^{-\frac{1}{2}}(4)$
 $= \frac{2}{\sqrt{4x - 1}}$

Hence, $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$
 $= (3x + 2) \times \frac{2}{\sqrt{4x - 1}} + \sqrt{4x - 1} \times 3$
 $= \frac{2(3x + 2)}{\sqrt{4x - 1}} + 3\sqrt{4x - 1}$
 $= \frac{2(3x + 2) + 3(4x - 1)}{\sqrt{4x - 1}}$

$$\frac{dy}{dx} = \frac{18x + 1}{\sqrt{4x - 1}}$$

<p>2. Diberi $y = \frac{2x+1}{x^2-3}$, cari $\frac{dy}{dx}$. <i>Given that $y = \frac{2x+1}{x^2-3}$, find $\frac{dy}{dx}$.</i></p> <p>Let $u = x$ and $v = \sqrt{x+3}$.</p> <p>Then, $\frac{dy}{dx} = x \frac{d}{dx} (\sqrt{x+3}) + \sqrt{x+3} \frac{d}{dx} (x)$</p> $= x \left(\frac{1}{2\sqrt{x+3}} \right) + \sqrt{x+3}$ $= \frac{x + 2(x+3)}{2\sqrt{x+3}}$ $\frac{dy}{dx} = \frac{3(x+2)}{2\sqrt{x+3}}$	<p>3. Diberi $y = \frac{2x+1}{x^2-3}$, tunjukkan bahawa $\frac{dy}{dx} = \frac{2x-1}{\sqrt{(4x-1)^3}}$. <i>Given that $y = \frac{2x+1}{x^2-3}$, show that $\frac{dy}{dx} = \frac{2x-1}{\sqrt{(4x-1)^3}}$.</i></p> $\frac{dy}{dx} = \frac{\sqrt{4x-1} \frac{d}{dx} (x) - x \frac{d}{dx} (\sqrt{4x-1})}{(\sqrt{4x-1})^2}$ $= \frac{\sqrt{4x-1} - \frac{2x}{\sqrt{4x-1}}}{4x-1}$ $= \frac{(\sqrt{4x-1})(\sqrt{4x-1}) - 2x}{(4x-1)\sqrt{4x-1}}$ $= \frac{4x-1-2x}{(4x-1)(\sqrt{4x-1})}$ $= \frac{(4x-1)(\sqrt{4x-1})}{(4x-1)(\sqrt{4x-1})}$ $\frac{dy}{dx} = \frac{2x-1}{\sqrt{(4x-1)^3}}$
--	--

Latihan Kendiri / Self Exercise

1. Cari $\frac{dy}{dx}$ bagi setiap fungsi berikut.
Find $\frac{dy}{dx}$ for each of the following function.

a) $y = 4x^2(5x + 3)$

b) $y = -2x^3(x + 1)$

$[60x^2 + 24x]$

$[8x^3 - 6x^2]$

<p>c) $y = x^2(1 - 4x)^4$</p> <p style="text-align: right;">$[2x(1-12x)(1 - 4x)^3]$</p>	<p>d) $y = x^2\sqrt{1 - 2x^2}$</p> <p style="text-align: right;">$[\frac{2x(1-3x^2)}{\sqrt{1-2x^2}}]$</p>
<p>e) $y = (4x - 3)(2x + 7)^6$</p> <p style="text-align: right;">$[8(7x-1)(2x + 7)^5]$</p>	<p>f) $y = (x + 5)^3(x - 4)^4$</p> <p style="text-align: right;">$[(7x+8)(x + 5)^2(x - 4)^3]$</p>

2. Bezakan setiap yang berikut terhadap x dengan menggunakan petua hasil darab.
Differentiate each of the following with respect to x by using product rule.

<p>a) $(1 - x^2)(6x + 1)$</p> <p style="text-align: right;">$[-2(9x^2 + x - 3)]$</p>
--

b) $\left(x + \frac{2}{x}\right)\left(x^2 - \frac{1}{x}\right)$

$$\left[3x^2 + 2 + \frac{4}{x^3}\right]$$

c) $(x^3 - 5)(x^2 - 2x + 8)$

$$[5x^4 - 8x^3 + 24x^2 - 10x + 10]$$

3. Diberi $f(x) = x\sqrt{x-1}$, cari nilai bagi $f'(5)$.
Given $f(x) = x\sqrt{x-1}$, find the value of $f'(5)$.

$$\left[\frac{13}{4}\right]$$

4. Cari kecerunan tangen bagi lengkung $y = x\sqrt{x^2 + 9}$ di $x = 4$.
Find the gradient of the tangent for the curve $y = x\sqrt{x^2 + 9}$ di $x = 4$.

$$\left[\frac{41}{5}\right]$$

5. Cari nilai pemalar r dengan keadaan $\frac{d}{dx}\left(\frac{2x-3}{x+5}\right) = \frac{r}{(x+5)^2}$
Find the value of constant r such that $\frac{d}{dx}\left(\frac{2x-3}{x+5}\right) = \frac{r}{(x+5)^2}$

[13]

6. Diberi $\frac{dy}{dx}$ bagi fungsi $y = ax^3 + bx^2 + 3$ pada titik (1,4) ialah 7, cari nilai a dan b .
Given $\frac{dy}{dx}$ for the function at the point (1,4) is 7. Find the values of a and b .

[$a = 5, b = -4$]

7. Diberi fungsi $h(x) = kx^3 - 4x^2 - 5x$, cari
Given function $h(x) = kx^3 - 4x^2 - 5x$, find
 a) $h'(x)$, dalam sebutan k ,
 $h'(x)$, in term of k
 b) nilai k jika $h'(1) = 8$
the value of k if $h'(1) = 8$

- a) $h'(x) = 3kx^2 - 8x - 5$
 b) 7

8. Jika $y = \frac{24}{(3x-5)^2}$, cari nilai bagi $\frac{dy}{dx}$ apabila $x = 2$.

If $y = \frac{24}{(3x-5)^2}$, find the value of $\frac{dy}{dx}$ when $x = 2$.

[-144]

9. Cari nilai bagi pemalar a dan pemalar b dengan keadaan $\frac{d}{dx} \left(\frac{1}{(3x-2)^3} \right) = -\frac{a}{(3x-2)^b}$

Find the value of constant a and constant b such that $\frac{d}{dx} \left(\frac{1}{(3x-2)^3} \right) = -\frac{a}{(3x-2)^b}$

[$a = 9, b = 4$]

10. Tunjukkan bahawa jika $f(x) = x\sqrt{x^2 + 3}$, maka $f'(x) = \frac{2x^2+3}{\sqrt{x^2+3}}$.

Show that if $f(x) = x\sqrt{x^2 + 3}$, then $f'(x) = \frac{2x^2+3}{\sqrt{x^2+3}}$.

2.3 Pembezaan Peringkat Kedua
The Second Derivative

2.3.1 Menentukan terbitan kedua bagi fungsi algebra
Second derivative of an algebraic function

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) \text{ atau/ or } f''(x) = \frac{d}{dx} [f'(x)]$$

Contoh 1:

Example 1:

(a) Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$ bagi fungsi $y = x^3 + \frac{4}{x^2}$.

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the function $y = x^3 + \frac{4}{x^2}$.

(b) Jika $g(x) = 2x^3 + 3x^2 - 7x - 9$, cari $g''(\frac{1}{4})$ dan $g''(-1)$.

If $g(x) = 2x^3 + 3x^2 - 7x - 9$, find $g''(\frac{1}{4})$ and $g''(-1)$.

Penyelesaian:

Solution:

$$\begin{aligned} \text{(a) } y &= x^3 + \frac{4}{x^2} \\ &= x^3 + 4x^{-2} \\ \frac{dy}{dx} &= 3x^2 - 8x^{-3} \\ \frac{d^2y}{dx^2} &= 6x + \frac{24}{x^4} \end{aligned}$$

$$\begin{aligned} \text{(b) } g(x) &= 2x^3 + 3x^2 - 7x - 9 \\ g'(x) &= 6x^2 + 6x - 7 \\ g''(x) &= 12x + 6 \\ \text{Maka } g''\left(\frac{1}{4}\right) &= 12\left(\frac{1}{4}\right) + 6 = 9 \\ g''(-1) &= 12(-1) + 6 = -6 \end{aligned}$$

Contoh 2:

Example 2:

Diberi fungsi $f(x) = x^3 + 2x^2 + 3x + 4$, cari nilai-nilai x dengan keadaan $f'(x) = f''(x)$.

Given the function $f(x) = x^3 + 2x^2 + 3x + 4$, find the values of x such that $f'(x) = f''(x)$.

Penyelesaian:

Solution:

Diberi $f(x) = x^3 + 2x^2 + 3x + 4$

Jadi, $f'(x) = 3x^2 + 4x + 3$ dan $f''(x) = 6x + 4$

$$f'(x) = f''(x)$$

$$3x^2 + 4x + 3 = 6x + 4$$

$$3x^2 - 2x - 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$$x = -\frac{1}{3} \text{ atau } x = 1$$

Maka, nilai-nilai x ialah $-\frac{1}{3}$ dan 1

Latihan Kendiri/ Self-Exercise:

1. Cari $\frac{dy}{dx}$ dan $\frac{d^2y}{dx^2}$ bagi setiap fungsi berikut.

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for each of the following functions.

$y = 3x^4 - 5x^2 + 2x - 1$ Jawapan/Answer: $\left[\frac{dy}{dx} = 12x^3 - 10x + 2 \right]$ $\frac{d^2y}{dx^2} = 36x^2 - 10]$	$y = 4x^2 - \frac{2}{x}$ Jawapan/Answer: $\left[\frac{dy}{dx} = 8x + \frac{2}{x^2} \right]$ $\frac{d^2y}{dx^2} = 8 - \frac{4}{x^3}]$	$y = (3x + 2)^8$ Jawapan/Answer: $\left[\frac{dy}{dx} = 24(3x + 2)^7 \right]$ $\frac{d^2y}{dx^2} = 504(3x + 2)^6]$
--	--	--

2. Cari $f'(x)$ dan $f''(x)$ bagi setiap fungsi berikut.
Find $f'(x)$ and $f''(x)$ for each of the following functions.

$f(x) = \sqrt{x} + \frac{1}{x^2}$ Jawapan/Answer: $\left[f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{x^3} \right]$ $f''(x) = -\frac{1}{4x^{\frac{3}{2}}} + \frac{6}{x^4}]$	$f(x) = \frac{x^4 + 2}{x^2}$ Jawapan/Answer: $\left[f'(x) = 2x - \frac{4}{x^3} \right]$ $f''(x) = 2 + \frac{12}{x^4}]$	$f(x) = \frac{2x + 5}{x - 1}$ Jawapan/Answer: $\left[f'(x) = -\frac{7}{(x-1)^2} \right]$ $f''(x) = \frac{14}{(x-1)^3}]$
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3. Diberi $y = x^3 + 3x^2 - 9x + 2$, cari koordinat titik A yang mungkin dengan keadaan $\frac{dy}{dx} = 0$. Seterusnya, cari nilai bagi $\frac{d^2y}{dx^2}$ di titik A itu.
Given $y = x^3 + 3x^2 - 9x + 2$, find the possible coordinates of A where $\frac{dy}{dx} = 0$. Then, find the value of $\frac{d^2y}{dx^2}$ at point A.

Jawapan/Answer: $[(-3, 29) \text{ dan } (1, -3) \quad x = -3, \frac{d^2y}{dx^2} = -12, x = 1, \frac{d^2y}{dx^2} = 12]$

Latihan Formatif/ Formative Exercise:

1. Jika $xy - 2x^2 = 3$, tunjukkan bahawa $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$.

If $xy - 2x^2 = 3$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$.

Jawapan/Answer: $[\frac{dy}{dx} = 2 - \frac{3}{x^2}, \frac{d^2y}{dx^2} = \frac{6}{x^3}]$

2. Cari $f'(1)$ dan $f''(1)$ bagi setiap fungsi berikut.

Find the value of $f'(1)$ and $f''(1)$ for each of the following functions.

$$f(x) = 3x - 2x^3$$

$$f(x) = x^2(5x - 3)$$

$$f(x) = \frac{x^3 + x}{x^2}$$

Jawapan/Answer:

$$[f'(1) = -3, f''(1) = -12]$$

Jawapan/Answer:

$$[f'(1) = 9, f''(1) = 24]$$

Jawapan/Answer:

$$[f'(1) = 0, f''(1) = 2]$$

3. Jika $f(x) = \sqrt{x^2 - 5}$ cari $f'(3)$ dan $f''(-3)$.

If $f(x) = \sqrt{x^2 - 5}$ find $f'(3)$ and $f''(-3)$.

Jawapan/Answer: $[f'(3) = \frac{3}{2}, f''(1) = \frac{-5}{8}]$

4. Jika $a = t^3 + 2t^2 + 3t + 4$, cari nilai-nilai t dengan keadaan $\frac{da}{dt} = \frac{d^2a}{dt^2}$.

If $a = t^3 + 2t^2 + 3t + 4$, find the values of t such that $\frac{da}{dt} = \frac{d^2a}{dt^2}$.

Jawapan/Answer: $[t = -\frac{1}{3}, t = 1]$

5. Diberi fungsi $g(x) = hx^3 - 4x^2 + 5x$. Cari nilai h jika $g''(1) = 4$.

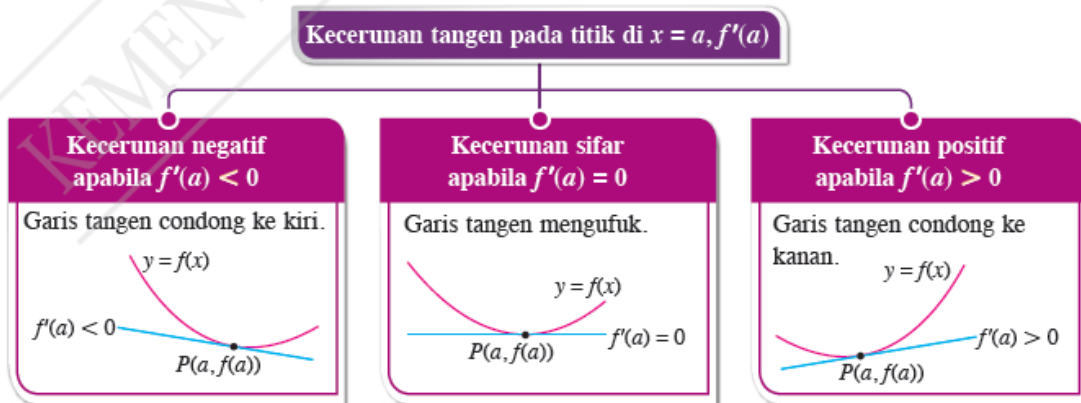
Given the function $g(x) = hx^3 - 4x^2 + 5x$. Find the value of h if $g''(1) = 4$.

Jawapan/Answer: $[h = 2]$

6. Diberi $f(x) = x^3 - x^2 - 8x + 9$, cari Given $f(x) = x^3 - x^2 - 8x + 9$, find	
(a) nilai-nilai x dengan keadaan $f'(x) = 0$ the values of x such that $f'(x) = 0$	(b) $f''(x)$
Jawapan/Answer: $[x = -\frac{4}{3}, x = 2]$	Jawapan/Answer: $[f''(x) = 6x - 2]$
(c) nilai x dengan keadaan $f''(x) = 0$ the value of x such as $f''(x) = 0$	(d) julat nilai x untuk $f''(x) < 0$ the range of x for $f''(x) < 0$
Jawapan/Answer:: $[x = \frac{1}{3}]$	Jawapan/Answer: $[x < \frac{1}{3}]$

2.4 Aplikasi Pembezaan
Application of Differentiation

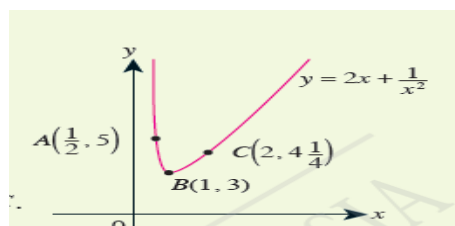
2.4.1 Kecerunan tangen kepada satu lengkung pada titik-titik yang berlainan
Gradient of tangent to a curve at different points



Contoh 1:
Example 1:

Rajah di sebelah menunjukkan sebahagian daripada lengkung $y = 2x + \frac{1}{x^2}$ dan titik-titik $A(\frac{1}{2}, 5)$, $B(1, 3)$ dan $C(2, 4\frac{1}{4})$ yang terletak pada lengkung itu.

The diagram on the right shows a part of the curve $y = 2x + \frac{1}{x^2}$ and the points $A(\frac{1}{2}, 5)$, $B(1, 3)$ and $C(2, 4\frac{1}{4})$ that are on the curve.



(a) Cari
Find

(i) ungkapan bagi $\frac{dy}{dx}$,
an expression for $\frac{dy}{dx}$,

(ii) kecerunan tangen bagi lengkung pada titik A, B dan C .
the gradient of the tangent to the curve at points A, B and C .

(b) Untuk setiap titik A, B dan C , nyatakan keadaan kecerunan tangennya pada lengkung itu.

For each of the points A, B and C , state the condition of the gradient of the tangent to the curve.

Penyelesaian:

Solution:

$$\begin{aligned} \text{a)(i) } y &= 2x + \frac{1}{x^2} \\ &= 2x + x^{-2} \\ \frac{dy}{dx} &= 2 - 2x^{-3} \\ &= 2 - \frac{2}{x^3} \end{aligned}$$

$$\text{(ii) Kecerunan tangen di } A \left(\frac{1}{2}, 5 \right) = 2 - \frac{2}{\left(\frac{1}{2}\right)^3} = -14$$

$$\text{Kecerunan tangen di } B(1, 3) = 2 - \frac{2}{(1)^3} = 0$$

$$\text{Kecerunan tangen di } C \left(2, 4\frac{1}{4} \right) = 2 - \frac{2}{(2)^3} = 1\frac{3}{4}$$

(b) Pada titik A , kecerunan tangennya ialah $-14 (< 0)$. Jadi kecerunannya adalah negatif dengan garis tangen condong ke kiri.

Pada titik B , kecerunan tangennya ialah 0 . Jadi kecerunannya adalah sifar dengan garis tangen adalah mengufuk.

Pada titik C , kecerunan tangennya ialah $1\frac{3}{4} (> 0)$. Jadi kecerunannya adalah positif dengan garis tangen condong ke kanan.

Latihan Kendiri/ Self-Exercise:

1. Persamaan bagi suatu lengkung ialah $y = 9x + \frac{1}{x}$ untuk $x > 0$.

The equation of a curve is $y = 9x + \frac{1}{x}$ for $x > 0$.

(a)(i) Cari kecerunan tangen kepada lengkung itu di $x = \frac{1}{4}$ dan $x = 1$.

Find the gradient of tangent to the curve at $x = \frac{1}{4}$ and $x = 1$.

(ii) Untuk setiap koordinat- x itu, nyatakan keadaan kecerunan tangennya kepada lengkung itu.

For each of the x -coordinates, state the condition of the gradient of the tangent to the curve.

(b) Seterusnya, cari koordinat titik pada lengkung dengan keadaan garis tangennya adalah mengufuk.

Subsequently, find the coordinates of the point where the tangent line is horizontal.

Jawapan/Answer:

[(a) (i) $-7,8$ (ii) Pada $x = \frac{1}{4}$, garis tangen condong ke kiri. Pada $x = 1$ pula, garis tangen condong ke kanan.

(b) $(\frac{1}{3}, 6), (-\frac{1}{3}, -6)$]

2. Lengkung $y = 9x^2 + \frac{b}{x}$ mempunyai kecerunan -14 dan 7 masing-masing di $x = \frac{1}{2}$ dan $x = 2$.

The curve $y = 9x^2 + \frac{b}{x}$ has gradients -14 and 7 at $x = \frac{1}{2}$ and $x = 2$ respectively.

(a) Tentukan nilai a dan nilai b .

Determine the values of a and b

(b) Cari koordinat titik pada lengkung dengan keadaan kecerunan tangennya ialah sifar.

Find the coordinates of the point on the curve where the gradient of the tangent is zero.

Jawapan/Answer:

(a) $[a = 2, b = 4]$ (b) $(1,6)$]

2.4.2 Persamaan tangen dan normal kepada satu lengkung pada suatu titik
The equation of tangent and normal to a curve at a point

Tangen dan normal

- Tangen: $y - f(a) = f'(a)(x - a)$
- Normal: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Tangent and normal

- Tangent: $y - f(a) = f'(a)(x - a)$
- Normal: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Contoh:

Example:

Cari persamaan tangen dan normal kepada lengkung $f(x) = x^3 - 2x^2 + 5$ pada titik $P(2,5)$.

Find the equation of the tangent and normal to the curve $f(x) = x^3 - 2x^2 + 5$ at point $P(2,5)$.

Penyelesaian:

Solution:

Diberi $f(x) = x^3 - 2x^2 + 5$, jadi $f'(x) = 3x^2 - 4x$.

Apabila $x = 2$, $f'(2) = 3(2)^2 - 4(2) = 4$

Kecerunan tangen pada titik $P(2,5)$ ialah 4.

Persamaan tangen ialah $y - 5 = 4(x - 2)$

$$y - 5 = 4x - 8$$

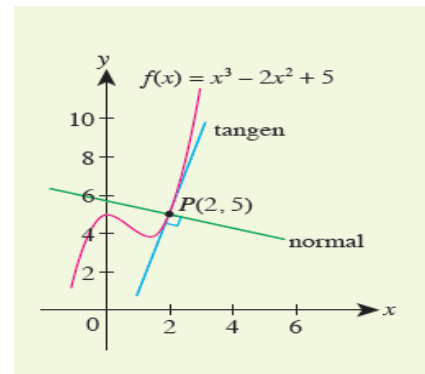
$$y = 4x - 3$$

Kecerunan normal pada titik $P(2,5)$ ialah $-\frac{1}{4}$.

Persamaan normal ialah $y - 5 = -\frac{1}{4}(x - 2)$

$$4y - 20 = -x + 2$$

$$4y = -x + 22$$



Latihan sendiri/ Self-Exercise:

1. Cari persamaan tangen dan normal kepada lengkung pada titik yang diberi berikut.

Find the equation of the tangent and normal to the following curves at the given points.

$f(x) = 5x^2 - 7x - 1$ pada titik $(1, -3)$

$f(x) = \sqrt{2x + 1}$ pada titik $(4, 3)$

$f(x) = \frac{x+1}{x-1}$ pada titik $(3, 2)$

Jawapan/Answer:

$[y = 3x - 6, 3y + x + 8 = 0]$

Jawapan/Answer:

$[3y - x = 5, y = -3x + 15]$

Jawapan/Answer:

$[2y = -x + 7, y = 2x - 4]$

<p>2. Cari persamaan tangen dan normal kepada lengkung pada titik yang diberi berikut. Find the equation of the tangent and normal to the following curves at the given value of x.</p>		
<p>$y = 2x^3 - 4x + 3, x = 1$</p> <p>Jawapan/Answer: [$y = 2x - 1, 2y + x = 3$]</p>	<p>$y = \sqrt{x} - \frac{1}{\sqrt{x}}, x = 4$</p> <p>Jawapan/Answer: [$16y - 5x = 4,$ $10y = -32x + 143$]</p>	<p>$y = \sqrt{x + 1}, x = 3$</p> <p>Jawapan/Answer: [$y = \frac{1}{4}x + \frac{5}{4}, y = -4x + 14$]</p>
<p>$y = \frac{5}{x^2+1}, x = -2$</p> <p>Jawapan/Answer: [$5y - 4x = 13, 4y + 5x + 6 = 0$]</p>	<p>$y = 2 + \frac{1}{x}, x = -1$</p> <p>Jawapan/Answer: [$y = -x, y = x + 2$]</p>	<p>$y = \frac{x^2+3}{x+1}, x = 3$</p> <p>Jawapan/Answer: [$y = \frac{3}{4}x + \frac{3}{4}, y = -\frac{4}{3}x + 7$]</p>
<p>3. Satu tangen dan normal dilukis pada lengkung $y = x\sqrt{1 - 2x}$ di $x = -4$. Cari A tangent and a normal is drawn to the curve $y = x\sqrt{1 - 2x}$ at $x = -4$. Find</p>		
<p>(a) nilai $\frac{dy}{dx}$ di $x = -4$ the value of $\frac{dy}{dx}$ at $x = -4$</p> <p>Jawapan/Answer: [$\frac{13}{3}$]</p>	<p>(b) Persamaan tangen the equation of the tangent</p> <p>Jawapan/Answer: [$3y - 13x = 16$]</p>	<p>(c) Persamaan normal the equation of the normal</p> <p>Jawapan/Answer: [$13y + 3x + 168 = 0$]</p>
<p>4(a) Tangen kepada lengkung $y = (x - 2)^2$ pada (3,1) melalui titik $(k, 7)$. Cari nilai k. The tangent to the curve $y = (x - 2)^2$ at the point (3,1) passes through $(k, 7)$. Find the value of k.</p> <p>(b) Normal kepada lengkung $y = 7x - \frac{6}{x}$ di $x = 1$ menyalang paksi-x di titik A. Cari koordinat A. The normal to the curve $y = 7x - \frac{6}{x}$ at $x = 1$ intersects the x-axis at A. Find the coordinates of A.</p> <p>Jawapan/Answer: [(a) 6 (b) A(14,0)]</p>		

2.4.3 Menyelesaikan masalah yang melibatkan tangen dan normal
Solving problems involving tangent and normal

Contoh:

Example:

Rajah di sebelah menunjukkan sebatang jalan raya yang boleh diwakili oleh lengkung

$y = \frac{1}{2}x^2 - 2x + 2$. Kumar memandu keretanya di jalan raya itu. Oleh kerana hujan, jalan tersebut menjadi licin dan menyebabkan Kumar tersasar di titik A lalu mengikut laluan AB yang merupakan garis tangen $y = 2x - c$ kepada jalan raya itu. Cari

The diagram on the right shows a road which is represented by the curve $y = \frac{1}{2}x^2 - 2x + 2$. Kumar drove on the road. As it was raining and the road was slippery, his car skidded at A and followed the line AB, which is tangent to the road at A and has an equation of $y = 2x - c$. Find

- (a) koordinat A
the coordinates of A.
- (b) nilai pemalar c.
the value of constant c.

Penyelesaian:

Solution:

(a) $y = \frac{1}{2}x^2 - 2x + 2$

$$\frac{dy}{dx} = x - 2$$

Oleh sebab $y = 2x - c$ ialah tangen kepada jalan raya $y = \frac{1}{2}x^2 - 2x + 2$ di titik A, jadi

$$\frac{dy}{dx} = 2$$

$$x - 2 = 2$$

$$x = 4$$

Oleh sebab titik A terletak di atas lengkung, jadi

$$y = \frac{1}{2}(4)^2 - 2(4) + 2$$

$$y = 2$$

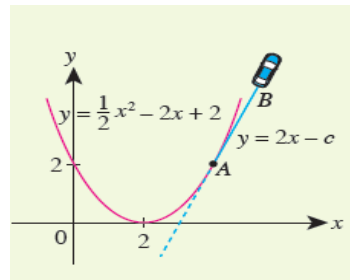
Maka, koordinat A ialah (4,2)

- (b) Titik A (4,2) terletak di atas laluan AB, iaitu $y = 2x - c$, jadi

$$2 = 2(4) - c$$

$$c = 6$$

Maka, nilai bagi pemalar c ialah 6.

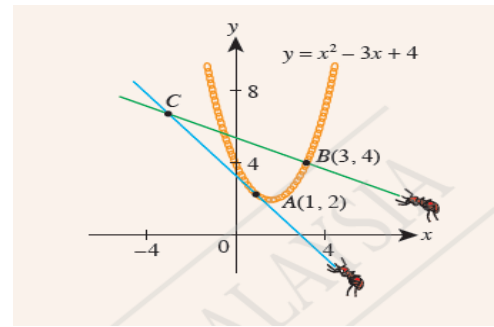


Latihan Kendiri/ Self-Exercise:

1. Rajah di sebelah menunjukkan seutas gelang tangan yang boleh diwakili oleh lengkung $y = x^2 - 3x + 4$ dengan keadaan titik A(1,2) dan titik B(3,4) terletak di atas gelang itu. Garis AC ialah tangen kepada gelang pada titik A dan garis BC pula ialah normal kepada gelang pada titik B. Dua ekor semut berjalan masing-masing di sepanjang garis tangen AC dan garis normal BC, dan bertemu pada titik C. Cari

The diagram on the right shows a bracelet which is represented by the curve $y = x^2 - 3x + 4$ where point A(1, 2) and point B(3, 4) are located on the bracelet. The line AC is a tangent to the bracelet at point A and the line BC is a normal to the bracelet at point B. Two ants move along AC and BC, and meet at point C. Find

- (a) persamaan tangen pada titik A
the equation of the tangent at point A
- (b) persamaan normal pada titik B
the equation of the normal at point B
- (c) koordinat C , iaitu titik pertemuan kedua-dua ekor semut.
the coordinates of C where the two ants meet.



Jawapan/Answer: [(a) $y + x = 3$, (b) $3y + x = 15$, (c) $C(-3,6)$]

2. Persamaan bagi suatu lengkung ialah $y = 2x^2 - 5x - 2$.

The equation of a curve is $y = 2x^2 - 5x - 2$

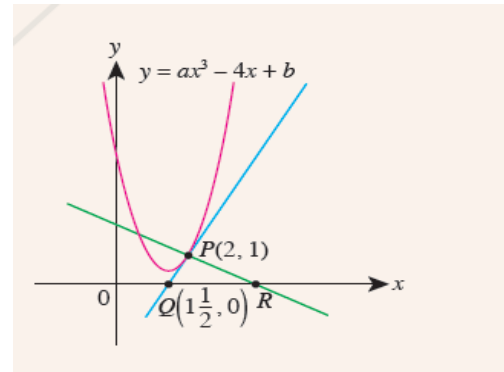
- (a) Cari persamaan normal kepada lengkung itu pada titik $A(1, -5)$.
Find the equation of a normal to the curve at point $A(1, -5)$.
- (b) Normal itu bertemu lengkung sekali lagi pada titik B . Cari koordinat B .
The normal meets the curve again at point B . Find the coordinates of B .
- (c) Seterusnya, cari koordinat titik tengah AB
Subsequently, find the coordinates of the midpoint of AB .

Jawapan/Answer: [(a) $y = x - 6$, (b) $B(2, -4)$, (c) $M_{AB} = (\frac{3}{2}, -\frac{9}{2})$]

3. Dalam rajah di sebelah, tangen kepada lengkung $y = ax^3 - 4x + b$ di $P(2,1)$ menyalang paksi-x di $Q(1\frac{1}{2}, 0)$. Normal di P pula menyalang paksi-x di R. Cari

In the diagram on the right, the tangent to the curve $y = ax^3 - 4x + b$ at $P(2,1)$ intersects the x-axis at the normal at P intersects the x-axis at R. Find

- (a) nilai a dan nilai b
the values of a and b
- (b) persamaan normal di titik P
the equation of the normal at point P
- (c) koordinat R
the coordinates of R
- (d) luas segi tiga PQR
the area of triangle PQR



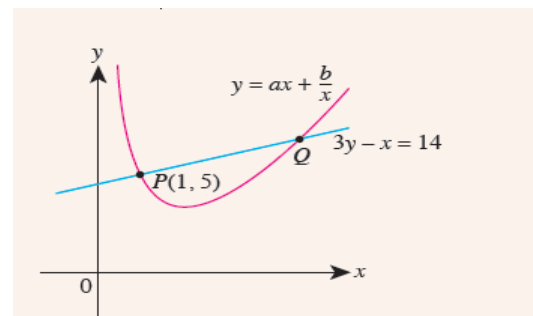
Jawapan/Answer: [(a) $a = \frac{1}{2}, b = 5$ (b) $2y + x = 4$, (c) $R = (4,0)$, (d) $1\frac{1}{4} \text{ unit}^2$]

4. Rajah di sebelah menunjukkan sebahagian daripada lengkung $y = ax + \frac{b}{x}$.

Garis $3y - x = 14$ adalah normal kepada lengkung di titik $P(1,5)$ dan normal ini bertemu lengkung sekali lagi di titik Q . Cari

The diagram on the right shows a part of the curve $y = ax + \frac{b}{x}$. The line $3y - x = 14$ is a normal to the curve at $P(1,5)$ and this normal intersects the curve again at Q. Find

- (a) nilai a dan nilai b ,
the values of a and b,
- (b) persamaan tangen di titik P ,
the equation of tangent at point P,
- (c) koordinat Q ,
the coordinates of Q.
- (d) koordinat titik tengah PQ .
the coordinates of the midpoint of PQ.



Jawapan/Answer: [(a) $a = 1, b = 4$ (b) $y + 3x = 8$, (c) $Q(6, 6\frac{2}{3})$, (d) $M_{PQ} = (3\frac{1}{2}, 5\frac{5}{6})$]

5. (a) Tangen kepada lengkung $y = \sqrt{2x + 1}$ di titik $A(4, 3)$ memotong paksi- x di titik B . Cari jarak AB .

The tangent to the curve $y = \sqrt{2x + 1}$ at point $A(4, 3)$ intersects the x -axis at point B . Find the distance of AB .

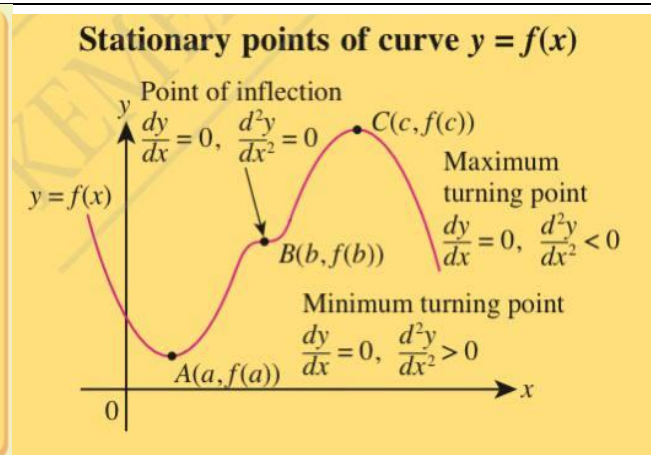
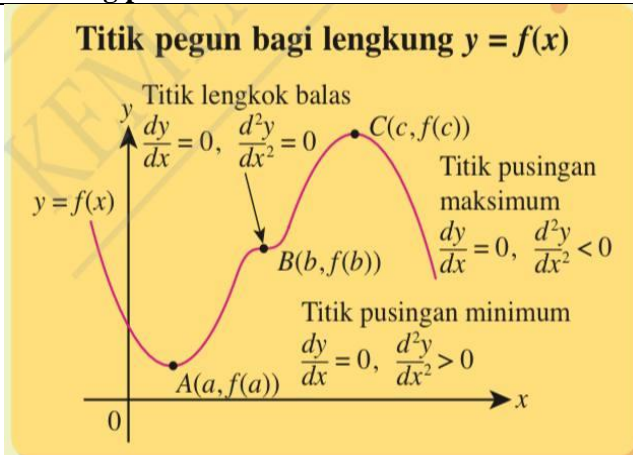
(b) Tangen kepada lengkung $y = hx^3 + kx + 2$ di $(1, \frac{1}{2})$ adalah selari dengan normal kepada lengkung di $(-2, -4)$. Cari nilai pemalar h dan nilai pemalar k .

The tangent to the curve $y = hx^3 + kx + 2$ at $(1, \frac{1}{2})$ is parallel to the normal to the curve. Find the value of constants h and k .

The tangent to the curve $y = hx^3 + kx + 2$ at $(1, \frac{1}{2})$ is parallel to the normal to the curve. Find the value of constants h and k .

Jawapan/Answer: [(a) $3\sqrt{10}$ unit (b) $h = \frac{1}{2}, k = -2$]

2.4.4 Titik pusingan dan sifat titik pusingan tersebut
Turning points and their nature



Contoh:

Example:

Diberi lengkung $y = x^3 - 3x^2 - 9x + 11$.

Given the curve $y = x^3 - 3x^2 - 9x + 11$.

(a) Cari koordinat titik pusingan bagi lengkung itu.

Find the coordinates of the turning points of the curve.

(b) Tentukan sama ada setiap titik pusingan itu ialah titik maksimum atau minimum.

Determine whether each of the turning points is a maximum or minimum point.

Penyelesaian:

Solution:

(a) $y = x^3 - 3x^2 - 9x + 11$

$$\frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x^2 - 2x - 9)$$

$$\frac{dy}{dx} = 3(x + 1)(x - 3)$$

Untuk titik pusingan, $\frac{dy}{dx} = 0$



$$x = -1 \text{ atau } x = 3$$

Apabila $x = -1, y = (-1)^3 - 3(-1)^2 - 9(-1) + 11$
 $y = 16$

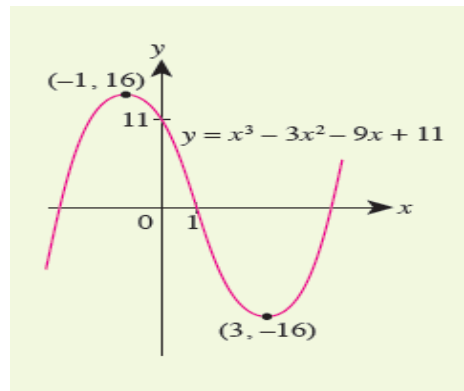
Apabila $x = 3, y = (3)^3 - 3(3)^2 - 9(3) + 11$
 $y = -16$

Maka, titik pusingan ialah $(-1, 16)$ dan $(3, -16)$.

(b)

x	-1.5	-1	-0.5	2.5	3	3.5
$\frac{dy}{dx}$	6.75	0	-5.25	-5.25	0	6.75
Tanda bagi $\frac{dy}{dx}$	+	0	-	-	0	+
Lakaran tangen	/	-	\	\	-	/
Lakaran graf						

Daripada jadual, tanda bagi $\frac{dy}{dx}$ berubah daripada positif kepada negative apabila x menokok melalui $x = -1$ dan tanda bagi $\frac{dy}{dx}$ berubah daripada negative kepada positif apabila x menokok melalui $x = 3$. Maka, titik pusingan $(-1, 16)$ ialah titik maksimum dan titik pusingan $(3, -16)$ ialah titik minimum.



Contoh:

Example:

Cari titik-titik pegun bagi lengkung $y = 2x^3 + 3x^2 - 12x + 5$ dan tentukan sifat setiap titik pegun itu.

Find the stationary points for each of the following curves and determine the nature of each stationary point

Penyelesaian:

Solution:

$$y = 2x^3 + 3x^2 - 12x + 5$$

$$\frac{dy}{dx} = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$$

$$\frac{dy}{dx} = 6(x + 2)(x - 1)$$

Untuk titik pusingan, $\frac{dy}{dx} = 0$

$$x = -2 \text{ atau } x = 1$$

Apabila $x = -2, y = 2(-2)^3 + 3(-2)^2 - 12(-2) + 5$

$$y = 25$$

Apabila $x = 1, y = 2(1)^3 + 3(1)^2 - 12(1) + 5$

$$y = -2$$

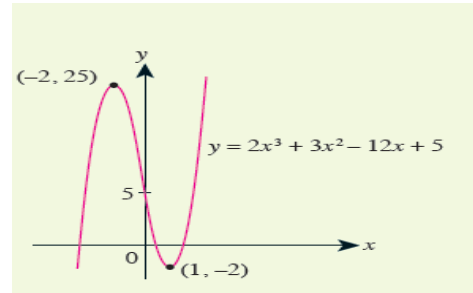
Maka, titik pusingan ialah $(-2, 25)$ dan $(1, -2)$.

$$\frac{d^2y}{dx^2} = 12x + 6$$

Apabila $x = -2, \frac{d^2y}{dx^2} = 12(-2) + 6 = -18 < 0$

Apabila $x = 1, \frac{d^2y}{dx^2} = 12(1) + 6 = 18 > 0$

Maka, $(-2, 25)$ ialah titik maksimum dan $(1, -2)$ ialah titik minimum.



Latihan Kendiri/ Self-Exercise:

1. Cari koordinat titik pusingan bagi setiap lengkung berikut. Dalam setiap kes, tentukan sama ada titik pusingan itu ialah titik maksimum atau titik minimum.

Find the coordinates of the turning points for each of the following curves. In each case, determine whether the turning points are maximum or minimum points.

$$y = x^3 - 12x$$

$$y = x(x - 6)^2$$

$$y = x\sqrt{18 - x^2}$$

Jawapan/Answer:

$(-2, 16)$ ialah titik maksimum
 $(2, -16)$ ialah titik minimum]

Jawapan/Answer:

$(2, 32)$ ialah titik maksimum
 $(6, 0)$ ialah titik minimum]

Jawapan/Answer:

$(3, 9)$ ialah titik maksimum
 $(-3, -9)$ ialah titik minimum]

$$y = (x - 6)(4 - 2x)$$

$$y = x + \frac{4}{x}$$

$$y = x^2 + \frac{1}{x^2}$$

<p>Jawapan/Answer: [(4,8) ialah titik maksimum]</p>	<p>Jawapan/Answer: [(1,2) ialah titik maksimum (2,4) ialah titik minimum]</p>	<p>Jawapan/Answer: [(0, -1) ialah titik maksimum (2,3) ialah titik minimum]</p>
<p>$y = x + \frac{1}{x-1}$</p> <p>Jawapan/Answer: [(0, -1) ialah titik maksimum (2,3) ialah titik minimum]</p>	<p>$y = \frac{(x-3)^2}{x}$</p> <p>Jawapan/Answer: [(-3, -12) ialah titik maksimum (3,0) ialah titik minimum]</p>	

2. Rajah di sebelah menunjukkan sebahagian daripada lengkung $y = x(x - 2)^3$.

The diagram on the right shows a part of the curve $y = x(x - 2)^3$.

(a) Cari ungkapan bagi $\frac{dy}{dx}$.

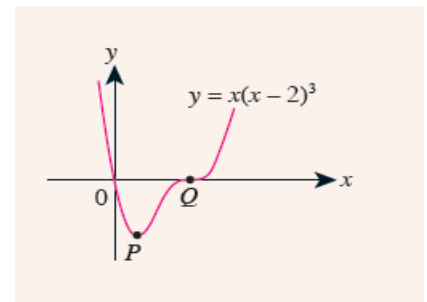
Find an expression for $\frac{dy}{dx}$

(b) Cari koordinat titik bagi dua titik pegun P dan Q .

Find the coordinates of the two stationary points, P and Q .

(c) Seterusnya, tentukan sifat bagi titik pegun Q menggunakan kaedah lakaran tangen.

Subsequently, determine the nature of stationary point Q by using the tangent sketching method.



Jawapan/Answer: [(a) $2(2x - 1)(x - 2)^2$ (b) $P\left(\frac{1}{2}, -\frac{27}{16}\right)$ dan $Q(2,0)$ (c) Q ialah titik lengkok balas

2.4.5 Menyelesaikan masalah yang melibatkan nilai maksimum dan nilai minimum serta mentafsir penyelesaian tersebut
Solving problems involving maximum and minimum values and interpreting the solutions

Contoh:

Example:

Sebuah kilang ingin menghasilkan tin makanan berbentuk silinder yang diperbuat daripada beberapa kepingan aluminium dengan isi padu 512 cm^3 . Permukaan melengkung tin dibentuk dengan menggulung sekeping aluminium berbentuk segi empat tepat manakala bahagian atas dan bawah tin dibentuk dengan memotong keluar dua buah bulatan daripada dua keping aluminium berbentuk segi empat sama. Cari jejari tapak tin itu, dalam cm , supaya jumlah luas permukaan semua kepingan aluminium yang digunakan adalah minimum.

A factory wants to produce cylindrical tins from aluminium sheets to contain food. Each tin has a volume of 512 cm^3 . The curved surface is made by rolling a rectangular piece of aluminium while the top and bottom are circular pieces cut out from two aluminium squares. Find the radius of the tin, in cm , such that the total surface of the aluminium sheets used will be minimum.

Penyelesaian:

Solution:

Isi padu tin, $\ell = 512$

$$\pi j^2 t = 512$$

$$t = \frac{512}{\pi j^2} \dots \dots \dots (1)$$

Jumlah luas permukaan, $L \text{ cm}^2$, kepingan-kepingan aluminium yang digunakan diberi oleh

$$L = 8j^2 + 2\pi jt \dots \dots \dots (2)$$

Gantikan (1) ke dalam (2)

$$L = 8j^2 + 2\pi j \left(\frac{512}{\pi j^2} \right)$$

$$L = 8j^2 + \frac{1024}{j}$$

$$\frac{dL}{dj} = 16j - \frac{1024}{j^2}$$

Untuk nilai minimum, $\frac{dL}{dj} = 0$

$$16j - \frac{1024}{j^2} = 0$$

$$16j^3 - 1024 = 0$$

$$j^3 = \frac{1024}{16}$$

$$j^3 = 64$$

$$j = 4$$

$$\frac{dL}{dj} = 16j - 1024j^{-2}$$

$$\frac{d^2L}{dj^2} = 16 + \frac{2048}{j^3}$$

Apabila $j = 4$, $\frac{d^2L}{dj^2} = 16 + \frac{2048}{4^3} = 48 > 0$

Maka, L mempunyai nilai minimum apabila jejari tapak ialah 4cm .

Latihan Kendiri/ Self- Exercise:

1. Seutas wayar dengan panjang 80 cm dibengkokkan untuk membentuk sebuah sektor POQ bagi sebuah bulatan berpusat O . Diberi bahawa $OQ = j$ cm dan $\angle POQ = \theta$ radian.

A wire of length 80 cm is bent to form a sector POQ of a circle with centre O . It is given that $OQ = r$ cm and $\angle POQ = \theta$ radian.

(a) Tunjukkan bahawa luas, A cm^2 , bagi sector POQ itu diberi oleh $A = \frac{1}{2}j(80 - 2j)$.

Show that the area, A cm^2 of the sector POQ is $A = \frac{1}{2}j(80 - 2j)$.

(b) Seterusnya, cari luas maksimum bagi sector POQ itu.

Then, find the maximum area of the sector POQ .

Jawapan/Answer: [400 cm^2]

2. Seutas dawai dengan panjang 240 cm dibengkokkan kepada suatu bentuk seperti yang ditunjukkan dalam rajah di sebelah.

A piece of wire of length 240 cm is bent to make a shape as shown in the diagram on the right.

(a) Ungkapkan y dalam sebutan x

Express y in terms of x .

(b) Tunjukkan bahawa luas, L cm^2 , yang dilitupi oleh dawai itu diberi oleh $L = 2880x - 540x^2$.

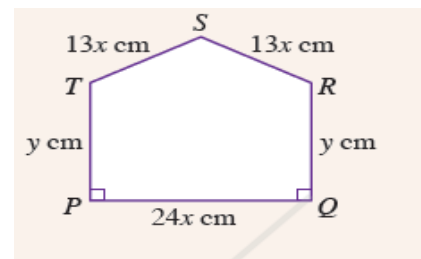
Show that the area, A cm^2 , enclosed by the wire is $A = 2880x - 540x^2$.

(c) Cari

Find

(i) nilai x dan nilai y supaya L adalah maksimum
the values of x and y for A to be maximum

(ii) luas maksimum, dalam cm^2 , rantau itu.
the maximum area enclosed by the wire in cm^2



Jawapan/Answer: [(a) $y = 120 - 25x$ (c)(i) $x = 2\frac{2}{3}$ cm, $y = 53\frac{1}{3}$ cm (ii) $3840cm^2$]

3. Sebuah kilang menghasilkan tin minuman berbentuk silinder tegak tertutup dengan isi padu $32\pi \text{ cm}^3$. Kos bahan yang digunakan untuk bulatan atas dan bawah tin itu ialah 2 sen per cm^2 manakala sisi melengkung tin ialah 1 sen per cm^2 .
- A factory produces cylindrical closed containers for drinks. Each container has a volume Of $32\pi \text{ cm}^3$. The cost of the material used to make the top and bottom covers of the container is 2 cents per cm^2 while the cost of the material to make the curved surface is 1cent per cm^2 .
- (a) Tunjukkan bahawa fungsi kos, C membuat tin minuman itu diberi oleh $C = 4\pi j^2 + \frac{64\pi}{j}$, dengan j ialah jejari tapak kon.
Show that the Cost, C to make a cylindrical drink container is $C = 4\pi j^2 + \frac{64\pi}{j}$, with r as the base radius of a cylinder.
- (b) Cari ukuran tin supaya kos yang digunakan oleh kilang itu adalah minimum.
Find the dimensions of each container produced in order for the cost to be minimum.

Jawapan/Answer: [(b) Jejari 2 cm dan tinggi 8cm]

2.4.6 Mentafsir dan menentukan kadar perubahan bagi kuantiti yang terhubung
Interpreting and determining rates of change for related quantities

Kadar perubahan yang terhubung
Jika dua pemboleh ubah yang terhubung x dan y berubah dengan masa, t , maka

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Rates of change of related quantities
If two variables, x and y change with time, t , then

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Contoh:

Example:

Suatu lengkung mempunyai persamaan $y = x^2 + \frac{4}{x}$. Cari

A curve has an equation $y = x^2 + \frac{4}{x}$. Find

- (a) ungkapan bagi $\frac{dy}{dx}$
an expression for $\frac{dy}{dx}$

- (b) kadar perubahan y apabila $x = 1$ dan $x = 2$, diberi bahawa x menokok dengan kadar tetap 3 unit per saat.
the rate of change of y when $x = 1$ and $x = 2$, given that x increases at a constant rate of 3 units per second.

Penyelesaian:**Solution:**

$$(a) y = x^2 + \frac{4}{x} = x^2 + 4x^{-1}$$

$$\frac{dy}{dx} = 2x - 4x^{-2} = 2x - \frac{4}{x^2}$$

$$(b) \text{Apabila } x = 1, \frac{dy}{dx} = 2(1) - \frac{4}{1^2} = -2$$

Kadar perubahan y diberi oleh :

$$\begin{aligned} \frac{dy}{dt} &= \frac{dy}{dx} \times \frac{dx}{dt} \\ &= -2 \times 3 \\ &= -6 \end{aligned}$$

Jadi, kadar perubahan dalam y ialah -6 unit per saat. Maka, y dikatakan menyusut pada kadar 6 unit per saat.

$$\text{Apabila } x = 2, \frac{dy}{dx} = 2(2) - \frac{4}{2^2} = 3$$

Kadar perubahan y diberi oleh :

$$\begin{aligned} \frac{dy}{dt} &= \frac{dy}{dx} \times \frac{dx}{dt} \\ &= 3 \times 3 \\ &= 9 \end{aligned}$$

Jadi, kadar perubahan dalam y ialah 9 unit per saat. Maka, y dikatakan menokok pada kadar 9 unit per saat.

Latihan Kendiri/ Self-Exercise:

1. Bagi setiap persamaan yang menghubungkan x dan y berikut, jika kadar perubahan x ialah 2 unit s^{-1} , cari kadar perubahan y pada ketika yang diberi.

For each of the following equations relating x and y , if the rate of change of x is 2 unit s^{-1} per second, find the rate of change of y at the given instant.

$$y = 3x^2 - 4, x = \frac{1}{2}$$

$$y = 2x^2 + \frac{1}{x}, x = 1$$

$$y = \frac{2}{(3x - 5)^3}, x = 2$$

Jawapan/Answer: [6 unit s^{-1}]

Jawapan/Answer:[6 unit s^{-1}]

Jawapan/Answer: [-36 unit s^{-1}]

$y = (4x - 3)^5, x = \frac{1}{2}$ <p>Jawapan/Answer: [40 unit s^{-1}]</p>	$y = \frac{x}{x + 1}, y = 2$ <p>Jawapan/Answer: [2 unit s^{-1}]</p>	$y = x^3 + 2, y = 10$ <p>Jawapan/Answer: [24 unit s^{-1}]</p>
<p>2. Bagi setiap persamaan yang menghubungkan x dan y berikut, jika kadar perubahan y ialah 6 unit s^{-1}, cari kadar perubahan x pada ketika yang diberi. For each of the following equations relating x and y, if the rate of change of y is 6 units per second, find the rate of change of x at the given instant.</p>		
$y = x^3 - 2x^2, x = 1$ <p>Jawapan/Answer: [-6 unit s^{-1}]</p>	$y = x^2 + \frac{4}{x}, x = 2$ <p>Jawapan/Answer: [2 unit s^{-1}]</p>	$y = \frac{2x^2}{x - 1}, x = 3$ <p>Jawapan/Answer: [4 unit s^{-1}]</p>
$y = (x - 6)\sqrt{x - 1}, x = 2$ <p>Jawapan/Answer: [-6 unit s^{-1}]</p>	$y = \frac{2x - 1}{x + 1}, y = 3$ <p>Jawapan/Answer: [18 unit s^{-1}]</p>	$y = \sqrt{2x + 7}, y = 3$ <p>Jawapan/Answer: [18 unit s^{-1}]</p>

3. Suatu lengkung mempunyai persamaan $y = (x - 8)\sqrt{x + 4}$. Cari

A curve has an equation $y = (x - 8)\sqrt{x + 4}$. Find

(a) ungkapan bagi $\frac{dy}{dx}$

an expression for $\frac{dy}{dx}$

(b) kadar perubahan y pada ketika $x = 5$, jika x menokok dengan kadar 6 unit per saat

the rate of change of y when $x = 5$ if x increases at a rate of 6 units per second.

Jawapan/Answer: [(a) $\frac{3x}{2\sqrt{x+4}}$ (b) 15 unit s^{-1}]

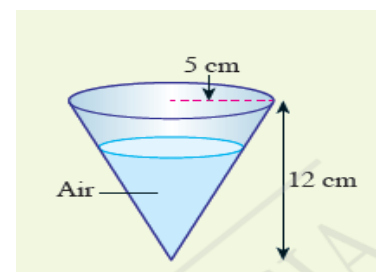
2.4.7 Menyelesaikan masalah yang melibatkan kadar perubahan bagi kuantiti yang terhubung dan mentafsir penyelesaian tersebut

Contoh:

Example:

Rajah di sebelah menunjukkan sebuah bekas berisi air yang berbentuk kon dengan jejari 5 cm dan tinggi 12 cm. Didapati bahawa air tersebut mengalir keluar melalui lubang kecil di hujung bekas dengan kadar tetap $4 \text{ cm}^3 \text{ s}^{-1}$. Cari kadar perubahan kedalaman air di dalam bekas itu apabila kedalaman air ialah 3 cm, betul kepada empat angka bererti.

The diagram on the right shows an inverted cone with a base radius of 5 cm and a height of 12 cm filled with some water. Water leaks out from a small hole at the tip of the cone at a constant rate of $4 \text{ cm}^3 \text{ s}^{-1}$. Find the rate of change of the depth of water in the cone when the height of water is 3 cm, correct to four significant figures.



Penyelesaian:

Solution:

Katakan r cm, h cm dan V cm³ masing-masing ialah jejari, tinggi dan isi padu air di dalam bekas itu pada masa t saat.

Jadi, $V = \frac{1}{3}\pi r^2 h$(1)

Didapati bahawa $\triangle DFE$ dan $\triangle BGE$ adalah serupa.

Jadi, $\frac{r}{5} = \frac{h}{12}$
 $r = \frac{5h}{12}$(2)

Gantikan (2) ke dalam (1)

$$V = \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{25h^3}{144}\right)$$

$$V = \frac{25\pi}{432} h^3$$

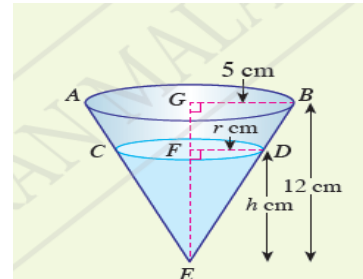
Kadar perubahan V diberi oleh petua rantai berikut.

$$\begin{aligned} \frac{dV}{dt} &= \frac{dV}{dh} \times \frac{dh}{dt} \\ &= \frac{d}{dh} \left(\frac{25\pi}{432} h^3 \right) \times \frac{dh}{dt} \\ \frac{dV}{dt} &= \frac{25\pi}{144} h^2 \times \frac{dh}{dt} \end{aligned}$$

Apabila $h = 3$ dan $\frac{dV}{dt} = -4$, kita peroleh

$$\begin{aligned} -4 &= \frac{25\pi}{144} (3)^2 \times \frac{dh}{dt} \\ -4 &= \frac{25\pi}{16} \times \frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{64}{25\pi} \\ &= -0.8148 \end{aligned}$$

Maka, kadar perubahan kedalaman air di dalam bekas itu ialah -0.8148cms^{-1} dan kedalaman air dikatakan menyusut pada kadar 0.8148cms^{-1} .



Contoh:

Jejari sebiji belon berbentuk sfera yang diisi dengan udara bertambah pada kadar tetap 0.5 cm per saat. Cari kadar perubahan isi padu belon itu apabila jejarinya ialah 4 cm, betul kepada empat angka bererti.

The radius of a spherical balloon filled with air increases at a rate of 0.5 cm per second. Find the rate of change of its volume when the radius is 4 cm, correct to four significant figures.

Penyelesaian:**Solution:**

Andaikan $l = f(j)$

Kadar perubahan l diberi oleh:

$$\frac{dl}{dt} = \frac{dl}{dj} \times \frac{dj}{dt}$$

Diketahui bahawa $l = \frac{4}{3}\pi j^3$

$$\text{Jadi, } \frac{dl}{dt} = \frac{d}{dj} \left(\frac{4}{3}\pi j^3 \right) \times \frac{dj}{dt}$$

$$\frac{dl}{dt} = 4\pi j^2 \times \frac{dj}{dt}$$

Apabila $j = 4$ dan $\frac{dj}{dt} = 0.5$, maka

$$\frac{dl}{dt} = 4\pi(4)^2 \times 0.5$$

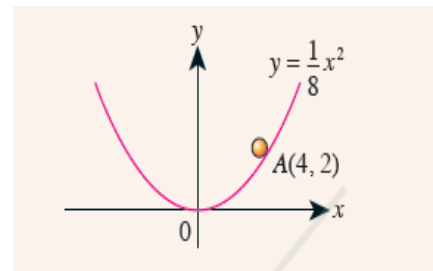
$$\frac{dl}{dt} = 4\pi(16) \times 0.5 = 32(3.142) = 100.5$$

Maka, kadar perubahan isi padu belon apabila $j = 4 \text{ cm}$ ialah 100.5 cm^3 per saat.

Latihan Kendiri/ Self-Exercise:

1. Rajah di sebelah menunjukkan sebutir manik yang bergerak di sepanjang lengkung $y = \frac{1}{8}x^2$. Pada titik $A(4,2)$, kadar perubahan x ialah 3 unit s^{-1} . Cari kadar perubahan y yang sepadan.

The diagram on the right shows a bead moving along a curve with the equation $y = \frac{1}{8}x^2$. At $A(4,2)$, the rate of change of x is 3 unit s^{-1} . Find the rate of change of the corresponding y .



Jawapan/Answer: [3 unit s^{-1}]

2. Luas sebuah segi empat sama dengan sisi $x \text{ cm}$ bertambah dengan kadar $8 \text{ cm}^2 \text{ s}^{-1}$. Cari kadar perubahan panjang sisinya apabila luasnya ialah 4 cm^2 .

The area of a square with side $x \text{ cm}$ increases at a rate of $8 \text{ cm}^2 \text{ s}^{-1}$. Find the rate of change of its side when the area is 4 cm^2 .

Jawapan/Answer: [2 cm s^{-1}]

3. Seketul ais berbentuk kubus dengan sisi $x \text{ cm}$ mencair pada kadar 10.5 cm^3 per minit. Cari kadar perubahan x pada Ketika $x = 10 \text{ cm}$.

A block of ice in the form of a cube with sides $x \text{ cm}$ is left to melt at a rate of 10.5 cm^3 per minute. Find the rate of change of x when $x = 10 \text{ cm}$.

Jawapan/Answer: $[-\frac{7}{200} \text{ cm min}^{-1}]$

4. Rajah di sebelah menunjukkan sebatang lilin yang berbentuk silinder tegak dan berjajari 3 cm . Tinggi lilin itu ialah $h \text{ cm}$ dan isi padunya ialah $V \text{ cm}^3$. Lilin itu terbakar dengan keadaan tingginya menyusut pada kadar 0.6 cm per minit.

The diagram on the right shows a cylindrical candle with radius 3 cm . The height is $h \text{ cm}$ and its volume is $V \text{ cm}^3$.

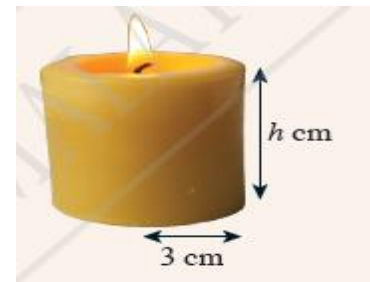
The candle is lit and the height decreases at a rate of 0.6 cm per minute.

(a) Ungkapkan V dalam sebutan h .

Express V in terms of h .

(b) Cari kadar perubahan isi padu lilin itu apabila tingginya ialah 8 cm .

Find the rate of change of the volume of the candle when its height is 8 cm .



Jawapan/Answer: $[(a)V = 9\pi h \quad (b) - 5.4\pi \text{ cm}^3 \text{ min}^{-1}]$

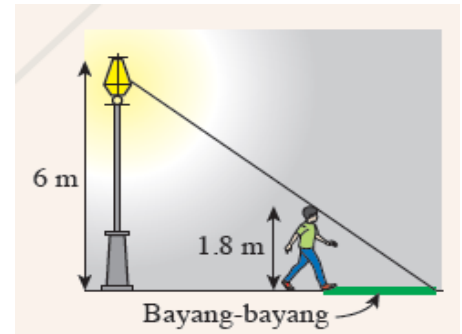
5. Chandran berjalan pada kadar 3.5 ms^{-1} daripada sebatang tiang lampu pada waktu malam seperti yang ditunjukkan dalam rajah di sebelah. Tinggi Chandran dan tiang lampu itu masing-masing ialah 1.8 m dan 6 m . Cari kadar perubahan

Chandran walks at a rate of 3.5 ms^{-1} away from a lamp post one night as shown in the diagram on the right. The heights of Chandran and the lamp post are 1.8 m and 6 m respectively. Find the rate of change of

(a) panjang bayang-bayang Chandran

Chandran's shadow

- (b) hujung bayang-bayangnya yang bergerak
the moving tip of the shadow



Jawapan/Answer: [(a) $1.5ms^{-1}$ (b) $5ms^{-1}$]

2.4.8 Mentafsir dan menentukan perubahan kecil dan penghampiran suatu kuantiti
Interpreting and determining small changes and approximations of certain quantities

<p>Perubahan kecil dan penghampiran Jika $y = f(x)$ dan perubahan kecil dalam x, iaitu δx menyebabkan perubahan kecil dalam y, iaitu δy, maka</p> $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $\delta y \approx \frac{dy}{dx} \times \delta x$ <p>dan $f(x + \delta x) \approx y + \frac{\delta y}{dx}$ $\approx y + \frac{dy}{dx} (\delta x)$</p>	<p>Small changes and approximations If $y = f(x)$ and the small change in x, that is δx, causes a small change in y, that is δy, then</p> $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ $\delta y \approx \frac{dy}{dx} \times \delta x$ <p>and $f(x + \delta x) \approx y + \frac{\delta y}{dx}$ $\approx y + \frac{dy}{dx} (\delta x)$</p>
--	--

Contoh:

Example:

Diberi bahawa $y = x^3$, cari

- (a) perubahan hamper dalam y jika x menokok daripada 4 kepada 4.05
(b) perubahan hamper dalam x jika y menyusut daripada 8 kepada 7.97

Penyelesaian:

Solution:

(a) $y = x^3$
 $\frac{dy}{dx} = 3x^2$
Apabila $x = 4$, $\delta x = 4.05 - 4 = 0.05$ dan
 $\frac{dy}{dx} = 3(4)^2 = 48$
Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x = 48 \times 0.05$
 $\delta y = 2.4$
Maka, perubahan hamper dalam y , iaitu δy ialah 2.4
 $\delta y > 0$ bermaksud berlakunya tokokan kecil dalam y sebanyak 2.4

(b) Apabila $y = 8$, $x^3 = 8$, $x = 2$
 $\delta y = 7.97 - 8 = -0.03$ dan
 $\frac{dy}{dx} = 3(2)^2 = 12$
Jadi, $\delta y \approx \frac{dy}{dx} \times \delta x$
 $-0.03 = 12 \times \delta x$
 $\delta x = \frac{-0.03}{12}$
 $\delta x = -0.0025$
Maka, perubahan hamper dalam x , iaitu δx ialah -0.0025
 $\delta x < 0$ bermaksud berlakunya susutan kecil dalam x sebanyak 0.0025

Contoh:

Example:

Diberi bahawa $y = \sqrt{x}$, cari

(a) nilai $\frac{dy}{dx}$ apabila $x = 4$

(b) nilai hampir bagi $\sqrt{4.02}$

Penyelesaian:

Solution:

(a) $y = \sqrt{x} = x^{\frac{1}{2}}$
 $\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
 Apabila $x = 4$, $\frac{dy}{dx} = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}$

(b) Apabila $x = 4$, $y = \sqrt{4} = 2$
 $\delta x = 4.02 - 4 = 0.02$ dan $\frac{dy}{dx} = \frac{1}{4}$
 Menggunakan $f(x + \delta x) \approx y + \frac{dy}{dx} \delta x$
 $\sqrt{4 + 0.02} = 2 + \frac{1}{4}(0.02)$
 $\sqrt{4.02} = 2.005$
 Maka, nilai hampir bagi $\sqrt{4.02}$ ialah 2.005

Contoh:

Example:

Diberi $y = 2x^2 - 3x + 4$. Apabila $x = 2$, terdapat perubahan kecil dalam x sebanyak 3%. Dengan menggunakan konsep kalkulus, cari peratus perubahan dalam y yang sepadan.

Penyelesaian:

Solution:

Diberi $y = 2x^2 - 3x + 4$
 Apabila $x = 2$, $y = 2(2)^2 - 3(2) + 4 = 6$
 $\frac{dy}{dx} = 4x - 3 = 4(2) - 3 = 5$ dan $\delta x = \frac{3}{100} \times 2 = 0.06$
 Jadi, $\delta y = \frac{dy}{dx} \times \delta x = 5 \times 0.06 = 0.3$
 $\frac{\delta y}{y} \times 100 = \frac{0.3}{6} \times 100 = 5$

Maka, peratus perubahan dalam y yang sepadan ialah 5%

Latihan Kendiri/ Self-Exercise:

- Beri setiap fungsi berikut, cari perubahan kecil dalam y yang sepadan dengan perubahan kecil dalam x yang diberi
 For each of the following functions, find the small corresponding change in y with the given small change in x .
 - $y = 4x^3 - 3x^2$, apabila x menokok daripada 1 kepada 1.05
 $y = 4x^3 - 3x^2$, when x increases from 1 to 1.05
 - $y = 4\sqrt{x} + 3x^2$ apabila x menyusut daripada 4 kepada 3.98
 $y = 4\sqrt{x} + 3x^2$, when x decreases from 4 to 3.98

Jawapan/Answer: [(a) 0.3 unit (b) -0.5 unit]

2. Bagi setiap fungsi berikut, cari perubahan kecil dalam x yang sepadan dengan perubahan kecil dalam y yang diberi.

For each of the following functions, find the small corresponding change in x with the given small change in y .

(a) $y = 2x^{\frac{3}{2}}$, apabila y menyusut daripada 16 kepada 15.7

$y = 2x^{\frac{3}{2}}$, when y decreases from 16 to 15.7

(b) $y = \frac{x+2}{2}$ apabila y menokok daripada 2 kepada $2 + p$

$y = \frac{x+2}{2}$ when y increases from 2 to $2 + p$

Jawapan/Answer: [(a) -0.05 unit (b) $2p$ unit]

3. Diberi $y = \frac{16}{x^2}$ cari nilai $\frac{dy}{dx}$ apabila $x = 2$ dan seterusnya tentukan nilai hampir bagi $\frac{16}{2.02^2}$

Given $y = \frac{16}{x^2}$ find the value of $\frac{dy}{dx}$ when $x = 2$ and determine the approximate value of $\frac{16}{2.02^2}$

Jawapan/Answer: [-4, 3.92]

4. Jika $y = x^{\frac{5}{4}}$, cari peritus perubahan hampir dalam x apabila terdapat 4% perubahan dalam y .
If $y = x^{\frac{5}{4}}$, find the approximate percentage change in x when there is 4% change in y .

Jawapan/Answer: [3.2%]

2.4.9 Menyelesaikan masalah yang melibatkan perubahan kecil dan penghampiran suatu kuantiti Solving problems involving small changes and approximations of certain quantities

Contoh:

Example:

Cari perubahan kecil dalam isi padu, $I \text{ cm}^3$, sebiji bola kaca yang berbentuk sfera apabila jejari, $j \text{ cm}$, bertambah daripada 3 cm kepada 3.02 cm .

Penyelesaian:

Solution:

Katakan $I \text{ cm}^3$ dan $j \text{ cm}$ masing-masing ialah isi padu dan jejari bola kaca itu.

$$\text{Jadi } I = \frac{4}{3}\pi j^3$$

$$\frac{dI}{dj} = 4\pi j^2$$

$$\text{Apabila } j = 3, \delta j = 3.02 - 3 = 0.02 \text{ dan } \frac{dI}{dj} = 4\pi(3)^2 = 36\pi$$

$$\text{Oleh itu, } \delta I = \frac{dI}{dj} \times \delta j = 36\pi \times 0.02$$

$$\delta I = 2.262$$

Maka, perubahan kecil dalam isi padu bola kaca itu ialah 2.262 cm^3

Latihan Kendiri/ Self-Exercise:

1. Tempoh ayunan, T saat, bagi suatu bandul dengan panjang $l \text{ cm}$ diberi oleh $T = 2\pi\sqrt{\frac{l}{10}}$ Cari perubahan hampir dalam T apabila l menokok daripada 9 cm kepada 9.05 cm

The period of oscillation, T second, of a pendulum with a length of $l \text{ cm}$ is given by $T = 2\pi\sqrt{\frac{l}{10}}$

Find the approximate change in T when l increases from 9 cm to 9.05 cm

Jawapan/Answer: $[\frac{\pi\sqrt{10}}{600} \text{ saat}]$

2. Luas tompokan minyak yang berbentuk bulatan bertambah dari $4\pi \text{ cm}^2$ kepada $4.01\pi \text{ cm}^2$. Cari perubahan kecil yang sepadan dalam jejari tompokan minyak itu.
The area of a drop of oil which spreads out in a circle increases from $4\pi \text{ cm}^2$ to $4.01\pi \text{ cm}^2$. Find the corresponding small change in the radius of the oil.

Jawapan/Answer: $[0.0025 \text{ cm}]$

3. Panjang sisi sebuah kubus ialah $x \text{ cm}$. Cari perubahan kecil dalam isi padu kubus itu apabila setiap sisinya menyusut daripada 2 cm kepada 1.99 cm .
The length of the side of a cube is $x \text{ cm}$. Find the small change in the volume of the cube when each side decreases from 2 cm to 1.99 cm .

Jawapan/Answer: $[-0.12 \text{ cm}^3]$

4. Cari perubahan kecil dalam isi padu sebuah sfera apabila jejaringnya menyusut daripada 5 cm kepada 4.98 cm .
Find the small change in the volume of a sphere when its radius decreases from 5 cm to 4.98 cm

Jawapan/Answer: $[-2\pi \text{ cm}^3]$

Soalan SPM 2003 K1 No 16

Diberi $y = x^2 + 5x$, gunakan kaedah pembezaan untuk mencari perubahan kecil bagi y apabila x menokok daripada 3 kepada 3.01

Given that $y = x^2 + 5x$, use differentiation to find the small change in y when x increases from 3 to 3.01

Jawapan/Answer: 0.11]

Soalan SPM 2003 K1 No 15

Diberi $y = 14x(5 - x)$, hitungkan

Given that $y = 14x(5 - x)$, calculate

- (a) nilai x apabila y adalah maksimum
the value of x when y is a maximum
- (b) nilai maksimum bagi y
the maximum value of y

Jawapan/Answer: [(a) $\frac{5}{2}$, (b) $\frac{175}{2}$]

Soalan SPM 2003 K2 No 3

- (a) Diberi $\frac{dy}{dx} = 2x + 2$ dan $y = 6$ apabila $x = -1$, carikan y dalam sebutan x .

Given that $\frac{dy}{dx} = 2x + 2$ and $y = 6$ when $x = -1$, find y in terms of x .

- (b) Seterusnya, carikan nilai x jika $x^2 \frac{d^2y}{dx^2} + (x - 1) \frac{dy}{dx} + y = 8$.

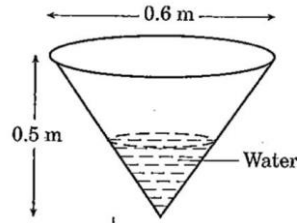
Hence, find the value of x if $x^2 \frac{d^2y}{dx^2} + (x - 1) \frac{dy}{dx} + y = 8$

Jawapan/Answer: [(a) $y = x^2 + 2x + 7$ (b) $x = \frac{3}{5}, x = -1$]

Soalan SPM 2003 K2 No 9

Rajah menunjukkan sebuah bekas berbentuk kon dengan diameter 0.6 m dan tinggi 0.5 m . Air dituangkan ke dalam bekas itu dengan kadar malar $0.2\text{ m}^3\text{ s}^{-1}$

Diagram shows a conical container of diameter 0.6 m and height 0.5 m . Water is poured into the container at a constant rate of $0.2\text{ m}^3\text{ s}^{-1}$



Hitungkan kadar perubahan tinggi paras air pada ketika tinggi paras air itu ialah 0.4 m

Calculate the rate of change of the height of the water level at the instant when the height of the water level is 0.4 m

(Gunakan $\pi = 3.142$, *isi padu kon* $= \frac{1}{3}\pi r^2 h$)

(Use $\pi = 3.142$, *Volume of cone* $= \frac{1}{3}\pi r^2 h$)

Jawapan/Answer: [1.105 m s^{-1}]

Soalan SPM 2004 K1 No 21

Dua pemboleh ubah, x dan y , dihubungkan oleh persamaan $y = 3x + \frac{2}{x}$.

Diberi bahawa y bertambah dengan kadar malar 4 unit sesaat, carikan kadar perubahan x apabila $x = 2$.

Two variables, x and y , are related by the equation $y = 3x + \frac{2}{x}$.

Given that y increases at a constant rate of 4 units per second, find the rate of change of x when $x = 2$.

Jawapan/Answer: [$\frac{8}{5}$ unit per second]

Soalan SPM 2004 K2 No 5

Fungsi kecerunan bagi suatu lengkung yang melalui $A(1, -12)$ ialah $3x^2 - 6x$. Carikan

The gradient function of a curve which passes through $A(1, -12)$ is $3x^2 - 6x$. Find

- (a) persamaan lengkung itu
the equation of the curve
- (b) koordinat titik-titik pusingan lengkung itu dan tentukan sama ada titik-titik pusingan itu adalah maksimum atau minimum.
the coordinates of the turning points of the curve and determine whether each of the turning points is a maximum or a minimum.

Jawapan/Answer: [(a) $y = 3x^2 - 6x - 10$, (b)(0, -10)titik maksimum, (2, -10)titik minimum]

Soalan SPM 2005 K1 No 19

Diberi $h(x) = \frac{1}{(3x-5)^2}$, nilaikan $h''(1)$

Given that $h(x) = \frac{1}{(3x-5)^2}$, evaluate $h''(1)$

Jawapan/Answer: $\left[\frac{27}{8}\right]$

Soalan SPM 2005 K1 No 20

Isi padu air, $V \text{ cm}^3$, dalam satu bekas diberi oleh $V = \frac{1}{3}h^3 + 8h$, dengan keadaan $h \text{ cm}$ ialah tinggi air dalam bekas itu. Air dituang ke dalam bekas itu dengan kadar $10 \text{ cm}^3 \text{ s}^{-1}$.

Carikan kadar perubahan tinggi air, dalam cm s^{-1} , pada ketika tingginya ialah 2 cm .

The volume of water, $V \text{ cm}^3$, in a container is given by $V = \frac{1}{3}h^3 + 8h$, where $h \text{ cm}$ is the height of the water in the container. Water is poured into the container at the rate of $10 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate of change of the height of water, in cm s^{-1} , at the instant when its height is 2 cm .

Jawapan/Answer: $\left[-\frac{5}{6} \text{ cms}^{-1}\right]$

Soalan SPM 2006 K1 No 18

Diberi bahawa $y = \frac{2}{3}u^7$, dengan keadaan $u = 3x - 5$. Cari $\frac{dy}{dx}$ dalam sebutan x .

It is given that $y = \frac{2}{3}u^7$, dengan keadaan $u = 3x - 5$. Find $\frac{dy}{dx}$ in terms of x .

Jawapan/Answer: $[14(3x - 5)^6]$

Soalan SPM 2006 K1 No 19

Diberi $y = 3x^2 + x - 4$,

Given that $y = 3x^2 + x - 4$,

(a) cari nilai bagi $\frac{dy}{dx}$ apabila $x = 1$.

find the value of $\frac{dy}{dx}$ when $x = 1$.

(b) ungkapkan perubahan kecil bagi y , dalam sebutan p , apabila x berubah daripada 1 kepada $1 + P$, dengan keadaan p ialah nilai yang kecil.

express the approximate change in y , in terms of p , when x changes from 1 to $1 + p$, where p is a small value.

Jawapan/Answer: $[(a)7, (b)7p]$

Soalan SPM 2007 K1 No 20

Lengkung $y = x^2 - 32x + 64$ mempunyai titik minimum di $x = p$, dengan p ialah pemalar. Cari nilai p .

The curve $y = x^2 - 32x + 64$ has a minimum point at $x = p$, where p is a constant. Find the value of p .

Jawapan/Answer: $[16]$

Soalan SPM 2007 K2 No 4

Suatu lengkung dengan fungsi kecerunan $2x - \frac{2}{x^2}$ mempunyai titik pusingan di $(k, 8)$.

A curve with gradient function $2x - \frac{2}{x^2}$ has a turning point at $(k, 8)$.

(a) Cari nilai k .

Find the value of k

(b) Tentukan sama ada titik pusingan ini adalah titik maksimum atau titik minimum.

Determine whether the turning point is a maximum or a minimum point.

(c) Cari persamaan lengkung itu.

Find the equation of the curve.

Jawapan/Answer: $[(a)k = 1, (b)(1,8) \text{ ialah titik minimum } (c)y = x^2 + \frac{2}{x} + 5]$

Soalan SPM 2008 K1 No 19

Dua pemboleh ubah, x dan y dihubungkan oleh persamaan $y = \frac{16}{x^2}$. Ungkapkan, dalam sebutan h , perubahan kecil bagi y , apabila x berubah daripada 4 kepada $4 + h$, dengan keadaan h ialah satu nilai kecil.

Two variables, x and y , are related by the equation $y = \frac{16}{x^2}$. Express, in terms of h , the approximate change in y , when x changes from 4 to $4 + h$, where h is a small value.

Jawapan/Answer: [16]

Soalan SPM 2008 K1 No 20

Garis normal kepada lengkung $y = x^2 - 5x$ pada titik P adalah selari dengan garis lurus $y = -x + 12$.

Cari persamaan garis normal kepada lengkung itu pada titik P .

The normal to the curve $y = x^2 - 5x$ at a point P is parallel to the straight line $y = -x + 12$.

Find the equation of the normal to the curve at point P .

Jawapan/Answer: $[y = -x - 3]$

Soalan SPM 2009 K1 No 19

Fungsi kecerunan suatu lengkung ialah $\frac{dy}{dx} = kx - 6$, dengan keadaan k ialah pemalar. Diberi bahawa lengkung itu mempunyai titik pusingan pada $(2,1)$

The gradient function of a curve is $\frac{dy}{dx} = kx - 6$, where k is a constant. It is given that the curve has a turning point at $(2,1)$

Cari

Find

(a) nilai k

the value of k

(b) persamaan lengkung itu

the equation of the curve.

Jawapan/Answer: $[(a)k = 3, (b)y = \frac{3}{2}x^2 - 6x + 7]$

Soalan SPM 2009 K1 No 21

Seketul ais berbentuk kubus dengan sisi x cm, mencair pada kadar 9.72cm^3 per minit. Cari kadar perubahan x pada ketika $x = 12$ cm.

A block of ice in the form of a cube with sides x cm, melts at a rate of 9.72cm^3 per minute. Find the rate of change of x at the instant when $x = 12$ cm.

Jawapan/Answer: $[-0.0225 \text{ cm min}^{-1}]$

Soalan SPM 2009 K2 No 3

Fungsi kecerunan suatu lengkung ialah $hx^2 - kx$, dengan keadaan h dan k ialah pemalar. Lengkung itu mempunyai titik pusingan pada $(3, -4)$. Kecerunan tangen kepada lengkung itu pada titik $x = 1$ ialah 8.

The gradient function of a curve is $hx^2 - kx$, where h and k are constants. The curve has a turning point at $(3, -4)$. The gradient of the tangent to the curve at the point $x = 1$ is 8.

Cari

Find

- (a) nilai h dan nilai k .
the value of h and of k .
- (b) persamaan lengkung itu.
the equation of the curve.

Jawapan/Answer: $\left[(a) h = 2, k = 6 \quad (b) y = \frac{2}{3}x^3 - 3x^2 + 5 \right]$

Soalan SPM 2010 K1 No 20

Diberi $y = 2x(x - 6)$, cari

Given $y = 2x(x - 6)$, find

- (a) $\frac{dy}{dx}$
- (b) nilai x apabila y adalah minimum
the value of x when y is minimum
- (c) nilai minimum bagi y
the minimum value of y

Jawapan/Answer: $\left[(a) \frac{dy}{dx} = 4x - 12 \quad (b) x = 3 \quad (c) -18 \right]$

Soalan SPM 2010 K1 No 21

Isi padu sebuah sfera bertambah dengan kadar tetap $12.8\pi \text{ cm}^3\text{s}^{-1}$. Cari jejari sfera itu pada Ketika jejari bertambah dengan kadar 0.2 cms^{-1} .

The volume of a sphere is increasing at a constant rate of $12.8\pi \text{ cm}^3\text{s}^{-1}$. Find the radius of the sphere at the instant when the radius is increasing at a rate of 0.2 cms^{-1} .

[Isi padu sfera, $V = \frac{4}{3}\pi r^3$]

[Volume of sphere, $V = \frac{4}{3}\pi r^3$]

Jawapan/Answer: [$r = 4 \text{ cm}$]

Soalan SPM 2010 K2 No 8

Lengkung $y = x^3 - 6x^2 + 9x + 1$ melalui titik $A(2,3)$ dan mempunyai dua titik pusingan, $P(3,1)$ dan Q .

The curve $y = x^3 - 6x^2 + 9x + 1$ passes through the point $A(2,3)$ and has two turning points, $P(3,1)$ and Q .

Cari

Find

(a) kecerunan lengkung itu pada A

the gradient of the curve at A .

(b) persamaan normal kepada lengkung itu pada A .

the equation of the normal to the curve at A .

(c) koordinat Q dan tentukan sama ada Q adalah titik maksimum atau titik minimum

the coordinates of Q and determine whether Q is the maximum or the minimum point.

Jawapan/Answer: [(a) -3 , (b) $x - 3y + 7 = 0$, (c) $(1,5)$, titik maksimum]

Soalan SPM 2011 K1 No 20

Diberi bahawa $y = 10 - \frac{12}{x}$. Cari perubahan kecil dalam x , dalam sebutan p , apabila nilai y berubah daripada 4 kepada $4 + P$.

It is given that $y = 10 - \frac{12}{x}$. Find the small change in x , dalam sebutan p , when the value of y changes from 4 to $4 + p$.

Jawapan/Answer: $\left[\frac{p}{3}\right]$

Soalan SPM 2012 K1 No 19

Diberi fungsi $h(x) = kx^3 - 4x^2 + 5x$, Cari

Given the function $h(x) = kx^3 - 4x^2 + 5x$, find

(a) $h'(x)$

(b) nilai k jika $h''(1) = 4$

the value of k if $h''(1) = 4$

Jawapan/Answer: [(a) $3kx^2 - 8x + 5$ (b) $k = 2$]

Soalan SPM 2012 K1 No 20

Kecerunan tangen kepada lengkung $y = x^2(2 + px)$ di $x = -2$ ialah 7.

Cari nilai p .

The gradient of the tangent to the curve $y = x^2(2 + px)$ at $x = -2$ is 7.

Find the value of p .

Jawapan/Answer: [$p = \frac{5}{4}$]

Soalan SPM 2012 K2 No 3

Suatu lengkung mempunyai fungsi kecerunan $kx - 6$, dengan keadaan k ialah pemalar. Diberi titik minimum bagi lengkung itu ialah $(3, -5)$, cari

A curve has a gradient function $kx - 6$, where k is a constant. Given that the minimum point of the curve is $(3, -5)$, find

(a) nilai k

the value of k

(b) pintasan-y bagi lengkung itu.

the y -intercept of the curve.

Jawapan/Answer: [(a) $k = 2$, (b) $c = 4$]

Soalan SPM 2013K1 No 19

Titik $P(1, -5)$ terletak pada lengkung $y = 3x^2 - 8x$.

The point $P(1, -5)$ lies on the curve $y = 3x^2 - 8x$.

Cari

Find

- (a) kecerunan tangen kepada lengkung itu di titik P
the gradient of the tangent to the curve at point P
- (b) persamaan normal kepada lengkung itu di titik P .
The equation of the normal to the curve at point P .

Jawapan/Answer: [(a) -2 (b) $2y = x - 11$]

Soalan SPM 2013 K1 No 20

Diberi $\frac{dv}{dt} = 8t - 3$ dan $v = 20$ apabila $t = 2$, ungkapkan v dalam sebutan t .

Given $\frac{dv}{dt} = 8t - 3$ and $v = 20$ when $t = 2$, express v in terms of t .

Jawapan/Answer: [$v = 4t^2 - 3t + c$]

Soalan SPM 2014 K1 No 17

Akibat daripada peningkatan kos sara hidup, Siva telah menanam beberapa jenis sayur untuk kegunaan sendiri di Kawasan lapang berbentuk segi empat tepat di belakang rumahnya. Dia bercadang untuk memagar Kawasan tersebut yang berukuran $6x$ m dan $(4 - x)$ m. Cari panjang, dalam m, pagar yang perlu dia beli apabila luas kawasan itu adalah maksimum.

Due to the high living cost, Siva has planted several types of vegetables for his own consumption on a rectangular shape empty plot of land behind his house. He plans to fence the land which has a dimension of $6x$ m and $(4 - x)$ m. Find the length, in m, the fence he has to buy when the area of the land is maximum.

Jawapan/Answer: [28 m]

Soalan SPM 2014 K1 No 18

Diberi $x = tx^2 + 3$ dan $\frac{dy}{dt} = 14t^3$, cari

Given $x = tx^2 + 3$ and $\frac{dy}{dt} = 14t^3$, find

- (a) $\frac{dy}{dt}$
 (b) $\frac{dy}{dx}$, dalam sebutan x
 $\frac{dy}{dx}$, in terms of x

Jawapan/Answer: [(a) $7t^2$ (b) $t^2 = x - 3$]

Soalan SPM 2015 K1 No 16

Fungsi kecerunan suatu lengkung ialah $(x - 3)$. Diberi $P(2,1)$ terletak pada lengkung itu.

The gradient function of a curve is $(x - 3)$. It is given $P(2,1)$ lies on the curve.

Cari

Find

- (a) kecerunan tangen pada titik P
 the gradient of the tangent at point P
 (b) the equation of the curve.
 persamaan lengkung itu.

Jawapan/Answer: [(a) -1 (b) $y = \frac{1}{2}x^2 - 3x + 5$]

Soalan SPM 2016 K1 No 6

Luas permukaan bagi sebuah kiub bertambah pada kadar tetap $15\text{cm}^2\text{s}^{-1}$. Cari kadar perubahan bagi panjang sisi, dalam cms^{-1} , ketika isi padu kiub ialah 125cm^3 .

The surface area of a cube increases at a constant rate of $15\text{cm}^2\text{s}^{-1}$. Find the rate of change of side length, in cms^{-1} , when the volume of the cube is 125cm^3 .

Jawapan/Answer: $\frac{1}{20}\text{cms}^{-1}$

Soalan SPM 2015 K2 No 2

Diberi persamaan suatu lengkung ialah $y = 2x(1 - x)^4$ dan lengkung itu melalui $P(2,4)$.

It is given the equation of the curve is $y = 2x(1 - x)^4$ and the curve passes through $P(2,4)$.

Cari

Find

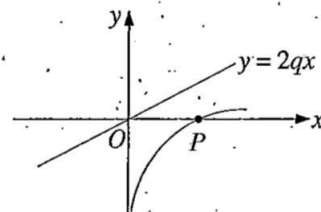
- (a) kecerunan lengkung pada titik P
the gradient of the curve at point P
- (b) persamaan garis normal kepada lengkung pada titik P
the equation of the normal to the curve at point P

Jawapan/Answer: [(a)18, (b) $x + 18y = 74$]

Soalan SPM 2016 K1 No 7

Rajah menunjukkan sebahagian daripada lengkung $y = \frac{2x-6}{x+2}$ dan satu garis lurus.

Diagram shows a part of curve $y = \frac{2x-6}{x+2}$ and a straight line.



Diberi bahawa garis lurus itu selari dengan tangen kepada lengkung itu pada titik P . Cari nilai q .

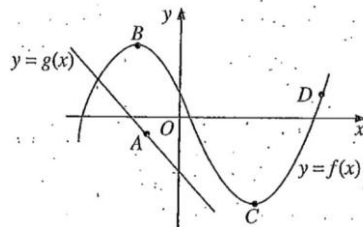
It is given that the straight line is parallel to the tangent of the curve at point P . Find the value of q .

Jawapan/Answer: $\left[q = \frac{1}{5} \right]$

Soalan SPM 2017 K1 No 1

Rajah 1 menunjukkan graf kubik $y = f(x)$ dan graf linear $y = g(x)$

Diagram 1 show a cubic graph $y = f(x)$ and a linear $y = g(x)$



A terletak pada garis lurus. Titik-titik B , C dan D terletak pada lengkung. Tangen kepada lengkung itu pada titik B dan titik C adalah selari dengan paksi x . Nyatakan titik-titik manakah yang memenuhi syarat berikut:

Point A lies on the straight line. Points B , C and D lie on the curve. The tangents to the curve at points B and C are parallel to the x -axis. State which point(s) that satisfies the following condition:

- (a) $\frac{dy}{dx} = 0$
- (b) $\frac{dy}{dx} < 0$
- (c) $\frac{d^2y}{dx^2} > 0$

Jawapan/Answer: [(a) B dan C (b) A (c) D]

Soalan SPM 2017 K2 No 2

Diberi bahawa persamaan suatu lengkung ialah $y = \frac{5}{x^2}$.

It is given that the equation of a curve is $y = \frac{5}{x^2}$.

- (a) Cari nilai $\frac{dy}{dx}$ apabila $x = 3$

Find the value of $\frac{dy}{dx}$ when $x = 3$

- (b) Seterusnya, anggarkan nilai bagi $\frac{5}{(2.98)^2}$.

Hence, estimate the value of $\frac{5}{(2.98)^2}$

Jawapan/Answer: [(a) $-\frac{10}{27}$ (b) 0.56296]

Soalan SPM 2018 K1 No 6

Diberi bahawa $L = 4t - t^2$ dan $x = 3 + 6t$.

It is given that $L = 4t - t^2$ and $x = 3 + 6t$.

(a) Ungkapkan $\frac{dL}{dx}$ dalam sebutan t .

Express $\frac{dL}{dx}$ dalam sebutan t .

(b) Cari perubahan kecil bagi x , apabila L berubah daripada 3 kepada 3.4 pada ketika $t = 1$.

Find the small change in x , when L changes from 3 to 3.4 at the instant $t = 1$.

Jawapan/Answer: [(a) $\frac{2-t}{3}$ (b) $\frac{6}{5}$]

Soalan SPM 2019 K1 No 17

Lengkung $y = px^4 + 2x$ mempunyai titik pusingan pada $(-1, q)$. Cari nilai p dan nilai q .

The curve $y = px^4 + 2x$ has turning point at $(-1, q)$. Find the value of p and of q .

Jawapan/Answer: [$p = \frac{1}{2}$, $q = \frac{-3}{2}$]

BAB 3 PENGAMIRAN
CHAPTER 3 INTEGRATION

3.1 Pengamiran sebagai Songsangan Pembezaan
Integration as the Inverse of Differentiation

3.1.1 Perkaitan antara pembezaan dengan pengamiran
The relation between differentiation and integration

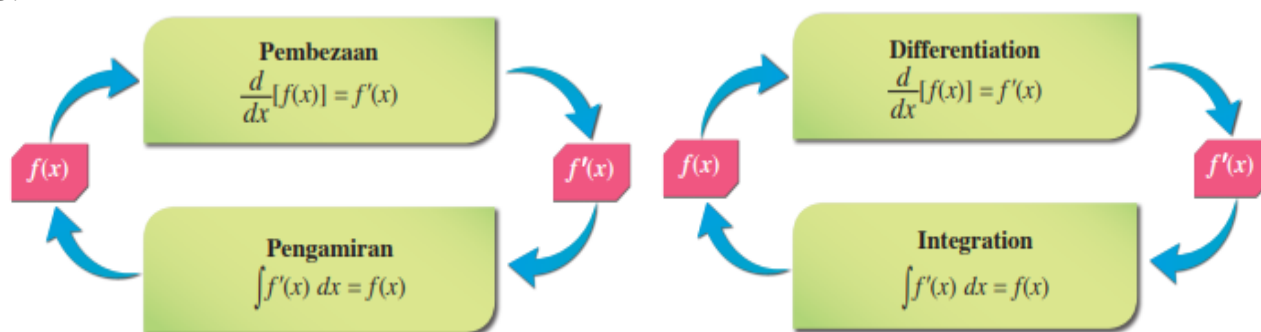
NOTA / NOTE :

1. Pengamiran ialah suatu proses songsangan bagi pembezaan.
Integration is in fact the reverse process of differentiation.

2. Jika y ialah satu fungsi x atau $y = f(x)$ dan $\frac{dy}{dx} = f'(x)$, maka $\int \frac{dy}{dx} dx = y + c$ atau $\int f'(x) dx = y + c$, dengan $c =$ pemalar.

If y is a function of x or $y = f(x)$ and $\frac{dy}{dx} = f'(x)$, then $\int \frac{dy}{dx} dx = y + c$ or $\int f'(x) dx = y + c$, such that $c =$ constant.

3.



4.

Jika $\frac{d}{dx}[f(x)] = f'(x)$, maka kamiran bagi $f'(x)$ terhadap x ialah $\int f'(x) dx = f(x)$.

If $\frac{d}{dx}[f(x)] = f'(x)$, then the integral of $f'(x)$ with respect to x is $\int f'(x) dx = f(x)$.

5. Imbas Kembali / Recall :

(a) Jika $y = ax^n$, maka $\frac{dy}{dx} = anx^{n-1}$. / (a) If $y = ax^n$, then $\frac{dy}{dx} = anx^{n-1}$.

(b) Jika $y = a$, maka $\frac{dy}{dx} = 0$. / (b) If $y = a$, then $\frac{dy}{dx} = 0$

Contoh 1:**Contoh 1**

Diberi $\frac{d}{dx}(4x^2) = 8x$, cari $\int 8x \, dx$.

Penyelesaian

Pembezaan bagi $4x^2$ ialah $8x$.

Secara songsangan, pengamiran bagi $8x$ ialah $4x^2$.

Oleh itu, $\int 8x \, dx = 4x^2$.

Example 1:**Example 1**

Given $\frac{d}{dx}(4x^2) = 8x$, find $\int 8x \, dx$.

Solution

Differentiation of $4x^2$ is $8x$.

By the reverse of differentiation, the integration of $8x$ is $4x^2$.

Hence, $\int 8x \, dx = 4x^2$.

Contoh 2:**Contoh 2**

Penghasilan arang batu di sebuah kawasan perlombongan diberi oleh $K = 48\,000t - 100t^3$, dengan keadaan K ialah jisim arang batu yang dihasilkan, dalam tan, dan t ialah masa, dalam tahun.

- (a) Cari kadar penghasilan arang batu, $\frac{dK}{dt}$, dalam sebutan t .
- (b) Jika kadar penghasilan arang batu berubah kepada $\frac{dK}{dt} = 96\,000 - 600t^2$, hitung jisim arang batu yang dihasilkan, dalam tan, pada tahun ke-4.

**Penyelesaian**

- (a) Diberi $K = 48\,000t - 100t^3$.

$$\text{Maka, } \frac{dK}{dt} = 48\,000 - 300t^2.$$

- (b) Diberi $\frac{dK}{dt} = 96\,000 - 600t^2$
 $= 2(48\,000 - 300t^2)$

Secara songsangan, pengamiran bagi $48\,000 - 300t^2$ ialah $48\,000t - 100t^3$.

$$\begin{aligned} \text{Oleh itu, } \int 2(48\,000 - 300t^2) \, dt &= 2(48\,000t - 100t^3) \\ &= 96\,000t - 200t^3 \end{aligned}$$

$$\begin{aligned} \text{Maka, jisim arang batu yang dihasilkan pada tahun ke-4} &= 96\,000(4) - 200(4)^3 \\ &= 371\,200 \text{ tan} \end{aligned}$$

Example 2:**Example 2**

The coal production from a coal mine is given by $K = 48\,000t - 100t^3$, where K is the mass of coal produced, in tonnes, and t is the time, in years.

- (a) Find the rate of production of coal, $\frac{dK}{dt}$, in terms of t .
- (b) If the rate of production of coal is given by $\frac{dK}{dt} = 96\,000 - 600t^2$, find the mass of coal produced, in tonnes, in the fourth year.

**Solution**

- (a) Given $K = 48\,000t - 100t^3$.

$$\text{Then, } \frac{dK}{dt} = 48\,000 - 300t^2.$$

- (b) Given $\frac{dK}{dt} = 96\,000 - 600t^2$
 $= 2(48\,000 - 300t^2)$

By the reverse of differentiation, the integration of $48\,000 - 300t^2$ is $48\,000t - 100t^3$.

$$\text{Hence, } \int 2(48\,000 - 300t^2) dt = 2(48\,000t - 100t^3) \\ = 96\,000t - 200t^3$$

$$\text{Therefore, the mass of coal produced in the fourth year} = 96\,000(4) - 200(4)^3 \\ = 371\,200 \text{ tonnes}$$

Latihan Kendiri 3.1 / Self-Exercise 3.1:

1. Diberi $\frac{d}{dx}(5x^3 + 4x) = 15x^2 + 4$, cari $\int (15x^2 + 4) dx$

Given $\frac{d}{dx}(5x^3 + 4x) = 15x^2 + 4$, find $\int (15x^2 + 4) dx$.

Jawapan/Answer: $5x^3 + 4x$

2. Diberi $\frac{d}{dx}(8x^3) = 24x^2$, cari $\int 24x^2 dx$

Given $\frac{d}{dx}(8x^3) = 24x^2$, find $\int 24x^2 dx$.

Jawapan/Answer: $8x^3$

3. Penggunaan air di sebuah pusat beli-belah A boleh diwakili oleh satu fungsi $J = 100t^3 + 30t^2$, dengan keadaan J ialah isipadu air yang digunakan, dalam liter, dan t ialah masa, dalam hari.

The usage of water at mall A is given by the function $J = 100t^3 + 30t^2$, where J is the volume of water used, in litres, and t is the time, in days.

- (a) Cari kadar penggunaan air bagi pusat beli-belah A, dalam sebutan t.

Find the rate of water used at mall A, in terms of t.

- (b) Jika kadar penggunaan air bagi pusat beli-belah A berubah kepada $\frac{dj}{dt} = 1500t^2 + 300t$, cari isipadu air, dalam liter, yang digunakan pada hari kedua.

If the rate of change of the water in mall A changes according to $\frac{dj}{dt} = 1500t^2 + 300t$, find the volume, in litres, used on the second day.

Jawapan/Answer: (a) $300t^2 + 60t$, (b) 4600 litres

Latihan Formatif 3.1 / Formative Exercise 3.1 :

1. Diberi $y = 3(2x+2)^3$, cari $\frac{dy}{dx}$. Seterusnya, cari $\int [18(2x+2)^2] dx$.

Given $y = 3(2x+2)^3$, find $\frac{dy}{dx}$. Subsequently, find $\int [18(2x+2)^2] dx$.

Jawapan/Answer: $18(2x+2)^2$, $3(2x+2)^3$

2. Diberi $y = \frac{5x+2}{2-3x}$, cari $f'(x)$ dan $\int f'(x) dx$.

Given $y = \frac{5x+2}{2-3x}$, find $f'(x)$ and $\int f'(x) dx$.

Jawapan/Answer: $\frac{16}{(2-3x)^2}$, $\frac{5x+2}{2-3x}$

3. Diberi $2x$ dan $\frac{dy}{dx} = h(x+2)^k$, cari nilai $h+k$. Seterusnya, cari nilai bagi $\frac{1}{10} \int \left(\frac{dy}{dx}\right) dx$ dengan keadaan $x = 2$.

Given $y = 5(x+2)^3$ and $\frac{dy}{dx} = h(x+2)^k$, find the value of $h+k$. Subsequently, find the value of $\frac{1}{10} \int \left(\frac{dy}{dx}\right) dx$ where $x = 2$.

Jawapan/Answer: 17,32

4. Diberi $f(x) = 3x(2x+1)^2$ dan $\int (12x^2 + 8x + 1) dx = af(x)$, cari nilai a .

Given $f(x) = 3x(2x+1)^2$ and $\int (12x^2 + 8x + 1) dx = af(x)$, find the values of a .

Jawapan/Answer: $\frac{1}{3}$

5. Fungsi keuntungan harian daripada jualan tiket bas bagi sebuah syarikat K diberi oleh $A = 100t^2 + 50t^3$, dengan keadaan A ialah keuntungan yang diperoleh, dalam RM, dan t ialah masa, dalam hari.

The profit function from the sale of bus tickets of company K is given by $A = 100t^2 + 50t^3$, where A is the profit obtained, in RM, and t is the time, in days.

(a) Kira kadar keuntungan jualan tiket bas yang diperoleh syarikat itu selepas 5 hari.
Find the rate of profit obtained by the bus company after 5 days.

(b) Diberi kadar keuntungan jualan tiket bas bagi sebuah syarikat H ialah $\frac{dA}{dt} = 30t^2 + 40t$, syarikat manakah yang

memperoleh keuntungan paling tinggi pada hari ke-10?

Given that the rate of profit obtained from another bus company H is given by $\frac{dA}{dt} = 30t^2 + 40t$, which company gets more profit on the 10th day?

Jawapan/Answer: (a) RM4750, (b) Syarikat K / Company K

3.2 Kamiran Tak Tentu Indefinite Integral

3.2.1 Rumus kamiran tak tentu Indefinite integral formula

NOTA / NOTE :

1. Rumus kamiran tak tentu terhadap x / Indefinite integral formula with respect to x :

(a)

• Bagi suatu pemalar a ,

$$\int a \, dx = ax + c, \text{ dengan keadaan } a \text{ dan } c \text{ ialah pemalar.}$$

• For a constant a ,

$$\int a \, dx = ax + c, \text{ where } a \text{ and } c \text{ are constants.}$$

(b)

• Bagi suatu fungsi ax^n ,

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c, \text{ dengan keadaan } a \text{ dan } c \text{ ialah pemalar, } n \text{ ialah integer dan } n \neq -1.$$

• For a function ax^n ,

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c, \text{ where } a \text{ and } c \text{ are constants, } n \text{ is an integer and } n \neq -1.$$

(c)

Kes 1

$$y = 5x, \frac{dy}{dx} = 5 \text{ dan} \\ \int 5 \, dx = 5x$$

Kes 2

$$y = 5x + 2, \frac{dy}{dx} = 5 \text{ dan} \\ \int 5 \, dx = 5x + 2$$

Kes 3

$$y = 5x - 3, \frac{dy}{dx} = 5 \text{ dan} \\ \int 5 \, dx = 5x - 3$$

$$\int 5 \, dx = 5x + c, \quad c = \text{pemalar pengamiran / constant of integration}$$

Case 1

$$y = 5x, \frac{dy}{dx} = 5 \text{ and} \\ \int 5 \, dx = 5x$$

Case 2

$$y = 5x + 2, \frac{dy}{dx} = 5 \text{ and} \\ \int 5 \, dx = 5x + 2$$

Case 3

$$y = 5x - 3, \frac{dy}{dx} = 5 \text{ and} \\ \int 5 \, dx = 5x - 3$$

(d)

Tip Pintar

Langkah-langkah untuk mencari kamiran ax^n terhadap x , dengan keadaan a ialah pemalar, n ialah integer dan $n \neq -1$:

1. Tambahkan indeks bagi x dengan 1.
2. Bahagikan sebutan dengan indeks baharu.
3. Tambahkan pemalar c dengan hasil kamiran.

Excellent Tip

Steps to find the integral of ax^n with respect to x , where a is a constant, n is an integer and $n \neq -1$:

1. Add 1 to the index of x .
2. Divide the term with the new index.
3. Add the constant c with the integrals.

Contoh 5:

Contoh 5

Cari kamiran bagi setiap yang berikut.

(a) $\int (3x^2 + 2) dx$

(b) $\int (x - 2)(x + 6) dx$

(c) $\int x^2 \left(3 + \frac{1}{x^5} \right) dx$

Penyelesaian

$$\begin{aligned} \text{(a)} \quad \int (3x^2 + 2) dx &= \int 3x^2 dx + \int 2 dx \\ &= \frac{3x^3}{3} + 2x + c \\ &= x^3 + 2x + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int (x - 2)(x + 6) dx &= \int (x^2 + 4x - 12) dx \\ &= \int x^2 dx + \int 4x dx - \int 12 dx \\ &= \frac{x^3}{3} + \frac{4x^2}{2} - 12x + c \\ &= \frac{x^3}{3} + 2x^2 - 12x + c \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int x^2 \left(3 + \frac{1}{x^5} \right) dx &= \int \left(3x^2 + \frac{1}{x^3} \right) dx \\ &= \int (3x^2 + x^{-3}) dx \\ &= \int 3x^2 dx + \int x^{-3} dx \\ &= \frac{3x^3}{3} + \frac{x^{-2}}{-2} + c \\ &= x^3 - \frac{1}{2x^2} + c \end{aligned}$$

PERBINCANGAN

Kamiran bagi suatu fungsi yang melibatkan penambahan dan penolakan sebutan-sebutan algebra boleh diwakilkan dengan satu pemalar pengamiran sahaja. Jelaskan.

Example 5:

Example 5

Find the integral for each of the following.

(a) $\int (3x^2 + 2) dx$

(b) $\int (x - 2)(x + 6) dx$

(c) $\int x^2 \left(3 + \frac{1}{x^5} \right) dx$

Solution

$$\begin{aligned} \text{(a)} \quad \int (3x^2 + 2) dx &= \int 3x^2 dx + \int 2 dx \\ &= \frac{3x^3}{3} + 2x + c \\ &= x^3 + 2x + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \int (x - 2)(x + 6) dx &= \int (x^2 + 4x - 12) dx \\ &= \int x^2 dx + \int 4x dx - \int 12 dx \\ &= \frac{x^3}{3} + \frac{4x^2}{2} - 12x + c \\ &= \frac{x^3}{3} + 2x^2 - 12x + c \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \int x^2 \left(3 + \frac{1}{x^5} \right) dx &= \int \left(3x^2 + \frac{1}{x^3} \right) dx \\ &= \int (3x^2 + x^{-3}) dx \\ &= \int 3x^2 dx + \int x^{-3} dx \\ &= \frac{3x^3}{3} + \frac{x^{-2}}{-2} + c \\ &= x^3 - \frac{1}{2x^2} + c \end{aligned}$$

DISCUSSION

Integration of functions containing algebraic terms added or subtracted together will have only one constant of integration. Discuss.

Latihan Kendiri 3.2 / Self-Exercise 3.2 :

1. Cari kamiran tak tentu bagi setiap yang berikut. / Find the indefinite integral for each of the following :

(a) $\int 2dx$

(b) $\int \frac{5}{6} dx$

(c) $\int -2dx$

(d) $\int \frac{\pi}{3} dx$

Jawapan/Answer: (a) $2x + c$

(b) $\frac{5}{6}x + c$

(c) $-2x + c$

(d) $\frac{\pi}{3}x + c$

2. Kamirkan setiap yang berikut terhadap x . / Integrate each of the following with respect to x :

(a) $\int 3x^2 dx$ (b) $\int \frac{4}{3} x^3 dx$ (c) $\int -x dx$ (d) $\int -\frac{2}{x^2} dx$
 (e) $\int \frac{3}{x^3} dx$ (f) $\int \frac{3}{\sqrt{x}} dx$ (g) $\int \frac{2}{\sqrt[3]{x}} dx$ (h) $\int \left(-\frac{3}{\sqrt{x}}\right)^3 dx$

Jawapan/Answer: (a) $x^3 + c$ (b) $\frac{x^4}{3} + c$ (c) $-\frac{x^2}{2} + c$ (d) $\frac{2}{x} + c$
 (e) $-\frac{3}{2x^2} + c$ (f) $2\sqrt{x^3} + c$ (g) $3\sqrt[3]{x^2} + c$ (h) $\frac{54}{\sqrt{x}} + c$

3. Kamirkan setiap yang berikut terhadap x . / Integrate each of the following with respect to x :

(a) $2x + 3$ (b) $4x^2 + 5x$ (c) $\frac{1}{2}x^3 + 5x - 2$ (d) $\frac{3}{x^2} + 4x - 2$

Jawapan/Answer: (a) $x^2 + 3x + c$ (b) $\frac{4}{3}x^3 + \frac{5}{2}x^2 + c$ (c) $\frac{1}{8}x^4 + \frac{5}{2}x^2 - 2x + c$ (d) $-\frac{3}{x} + 2x^2 - 2x + c$

4. Cari kamiran tak tentu bagi setiap yang berikut. / Find the indefinite integral for each of the following :

(a) $\int (x+2)(x-4)dx$

(b) $\int x^2(3x^2+5x)dx$

(c) $\int (5x^2-3\sqrt{x})dx$

(d) $\int (5x-3)^2 dx$

(e) $\int \left(\frac{5x^2-3x}{x} \right) dx$

(f) $\int (x+\sqrt{x})^2 dx$

Jawapan/Answer: (a) $\frac{x^3}{3} - x^2 - 8x + c$

(b) $\frac{3}{5}x^5 + \frac{5}{4}x^4 + c$

(c) $\frac{5}{3}x^3 - 2\sqrt{x^3} + c$

(d) $\frac{25}{3}x^3 - 15x^2 + 9x + c$

(e) $\frac{5}{2}x^2 - 3x + c$

(f) $\frac{1}{3}x^3 + \frac{4}{5}x^{\frac{5}{2}} + \frac{1}{2}x^2 + c$

3.2.3 Kamiran tak tentu bagi fungsi berbentuk $(ax+b)^n$, dengan keadaan a dan b ialah pemalar, n ialah integer dan $n \neq -1$

Indefinite integral for functions in the form of $(ax+b)^n$, where a and b are constants, n is an integer and $n \neq -1$

NOTA / NOTE :

Terbitan rumus kamiran tak tentu bagi fungsi berbentuk $(ax+b)^n$, dengan keadaan a dan b ialah pemalar, n ialah integer dan $n \neq -1$:

Derivation of indefinite integral formula for functions in the form of $(ax+b)^n$, where a and b are constants, n is an integer and $n \neq -1$:

Jika diberi fungsi $y = \int (ax+b)^n dx$, maka $\frac{dy}{dx} = (ax+b)^n$.

If given function $y = \int (ax+b)^n dx$, then $\frac{dy}{dx} = (ax+b)^n$



Imbas Kembali

Bagi suatu fungsi

$$y = g(u) \text{ dan } u = h(x),$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Katakan, $u = ax + b$

Jadi, $\frac{du}{dx} = a$

dan $\frac{dy}{dx} = u^n$

Dengan menggunakan petua rantai,

$$\begin{aligned} \frac{dy}{du} &= \frac{dy}{dx} \times \frac{dx}{du} \\ &= \frac{dy}{dx} \times \frac{1}{\left(\frac{du}{dx}\right)} \end{aligned}$$

Gantikan $\frac{dy}{dx} = u^n$ dan $\frac{du}{dx} = a$, kita peroleh

$$\frac{dy}{du} = u^n \times \frac{1}{a}$$

$$y = \int \frac{u^n}{a} du$$

$$\begin{aligned} \int (ax + b)^n dx &= \int \frac{u^n}{a} du \\ &= \frac{1}{a} \int u^n du \\ &= \frac{1}{a} \left[\frac{u^{n+1}}{n+1} \right] + c \end{aligned}$$

Gantikan $u = ax + b$, kita peroleh

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, \text{ dengan keadaan } a \text{ dan } b \text{ ialah pemalar, } n \text{ ialah integer dan } n \neq -1.$$



Recall

For a function $y = g(u)$

and $u = h(x)$,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Let $u = ax + b$

Then, $\frac{du}{dx} = a$

and $\frac{dy}{dx} = u^n$

With chain rule,

$$\begin{aligned} \frac{dy}{du} &= \frac{dy}{dx} \times \frac{dx}{du} \\ &= \frac{dy}{dx} \times \frac{1}{\left(\frac{du}{dx}\right)} \end{aligned}$$

Substitute $\frac{dy}{dx} = u^n$ and $\frac{du}{dx} = a$, and we get

$$\frac{dy}{du} = u^n \times \frac{1}{a}$$

$$y = \int \frac{u^n}{a} du$$

$$\begin{aligned} \int (ax + b)^n dx &= \int \frac{u^n}{a} du \\ &= \frac{1}{a} \int u^n du \\ &= \frac{1}{a} \left[\frac{u^{n+1}}{n+1} \right] + c \end{aligned}$$

Substitute $u = ax + b$, and we get

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$$

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c, \text{ where } a \text{ and } b \text{ are constants, } n \text{ is an integer and } n \neq -1.$$

Contoh 6:

Contoh 6

Dengan menggunakan kaedah penggantian, cari kamiran tak tentu bagi setiap yang berikut.

(a) $\int (3x + 5)^5 dx$

(b) $\int \sqrt{5x + 2} dx$

Penyelesaian

(a) Katakan $u = 3x + 5$

Jadi, $\frac{du}{dx} = 3$

$$dx = \frac{du}{3}$$

$$\begin{aligned} \int (3x + 5)^5 dx &= \int \frac{u^5}{3} du \\ &= \frac{1}{3} \left(\frac{u^6}{6} \right) + c \\ &= \frac{(3x + 5)^6}{18} + c \end{aligned}$$

(b) Katakan $u = 5x + 2$

Jadi, $\frac{du}{dx} = 5$

$$dx = \frac{du}{5}$$

$$\begin{aligned} \int \sqrt{5x + 2} dx &= \int \frac{\sqrt{u}}{5} du \\ &= \int \frac{u^{\frac{1}{2}}}{5} du \\ &= \frac{2}{15} u^{\frac{3}{2}} + c \\ &= \frac{2}{15} (5x + 2)^{\frac{3}{2}} + c \end{aligned}$$

Example 6:

Example 6

By using substitution method, find the indefinite integral for each of the following.

(a) $\int (3x + 5)^5 dx$

(b) $\int \sqrt{5x + 2} dx$

Solution

(a) Let $u = 3x + 5$

Then, $\frac{du}{dx} = 3$

$$dx = \frac{du}{3}$$

$$\begin{aligned} \int (3x + 5)^5 dx &= \int \frac{u^5}{3} du \\ &= \frac{1}{3} \left(\frac{u^6}{6} \right) + c \\ &= \frac{(3x + 5)^6}{18} + c \end{aligned}$$

(b) Let $u = 5x + 2$

Then, $\frac{du}{dx} = 5$

$$dx = \frac{du}{5}$$

$$\begin{aligned} \int \sqrt{5x + 2} dx &= \int \frac{\sqrt{u}}{5} du \\ &= \int \frac{u^{\frac{1}{2}}}{5} du \\ &= \frac{2}{15} u^{\frac{3}{2}} + c \\ &= \frac{2}{15} (5x + 2)^{\frac{3}{2}} + c \end{aligned}$$

Contoh 7:**Contoh 7**

Kamirkan setiap yang berikut terhadap x .

(a) $(2 - 3x)^4$

(b) $\frac{3}{(5x - 3)^6}$

Penyelesaian

$$\begin{aligned} \text{(a)} \int (2 - 3x)^4 dx &= \frac{(2 - 3x)^5}{-3(5)} + c \\ &= -\frac{(2 - 3x)^5}{15} + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \int \frac{3}{(5x - 3)^6} dx &= \int 3(5x - 3)^{-6} dx \\ &= \frac{3(5x - 3)^{-5}}{5(-5)} + c \\ &= -\frac{3}{25(5x - 3)^5} + c \end{aligned}$$

Example 7:**Example 7**

Integrate each the following with respect to x .

(a) $(2 - 3x)^4$

(b) $\frac{3}{(5x - 3)^6}$

Solution

$$\begin{aligned} \text{(a)} \int (2 - 3x)^4 dx &= \frac{(2 - 3x)^5}{-3(5)} + c \\ &= -\frac{(2 - 3x)^5}{15} + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \int \frac{3}{(5x - 3)^6} dx &= \int 3(5x - 3)^{-6} dx \\ &= \frac{3(5x - 3)^{-5}}{5(-5)} + c \\ &= -\frac{3}{25(5x - 3)^5} + c \end{aligned}$$

Latihan kendiri 3.3 / Self-Exercise 3.3 :

1. Cari kamiran tak tentu bagi setiap yang berikut dengan menggunakan kaedah penggantian.

Find the indefinite integral for each of the following by using substitution method.

(a) $\int (x - 3)^2 dx$

(b) $\int (3x - 5)^9 dx$

(c) $\int 4(5x - 2)^5 dx$

Jawapan/Answer:

$$\frac{(x - 3)^3}{3} + c$$

Jawapan/Answer:

$$\frac{(3x - 5)^{10}}{30} + c$$

Jawapan/Answer:

$$\frac{2}{15}(5x - 2)^6 + c$$

$$(d) \int \frac{(7x-3)^4}{3} dx$$

Jawapan/Answer:

$$\frac{(7x-3)^5}{105} + c$$

$$(e) \int \frac{12}{(2x-6)^3} dx$$

Jawapan/Answer:

$$-\frac{3}{(2x-6)^2} + c$$

$$(f) \int \frac{2}{3(3x-2)^2} dx$$

Jawapan/Answer:

$$-\frac{2}{9(3x-2)} + c$$

2. Kamirkan setiap yang berikut terhadap x.
Integrate each of the following with respect to x.

$$(a) (4x+5)^4$$

Jawapan/Answer:

$$\frac{(4x+5)^5}{20} + c$$

$$(b) 2(3x-2)^3$$

Jawapan/Answer:

$$\frac{(3x-2)^4}{6} + c$$

$$(c) (5x-11)^4$$

Jawapan/Answer:

$$\frac{(5x-11)^5}{25} + c$$

$$(d) \frac{(3x-2)^5}{5}$$

Jawapan/Answer:

$$\frac{(3x-5)^6}{90} + c$$

$$(e) \frac{5}{(6x-3)^6}$$

Jawapan/Answer:

$$-\frac{1}{6(6x-3)^5} + c$$

$$(f) \frac{12}{(3x-5)^8}$$

Jawapan/Answer:

$$-\frac{4}{7(3x-5)^7} + c$$

3.2.4 Persamaan lengkung daripada fungsi kecerunan

Equation of a curve from its gradient function

NOTA / NOTE :

1.

Diberi suatu fungsi kecerunan $\frac{dy}{dx} = f'(x)$, maka persamaan lengkung bagi fungsi itu ialah $y = \int f'(x) dx$.

Given the gradient function $\frac{dy}{dx} = f'(x)$, the equation of curve for that function is $y = \int f'(x) dx$.

2. Nilai pemalar pengamiran, c boleh ditentukan dengan menggantikan nilai x dan y yang sepadan ke dalam hasil pengamiran suatu fungsi kecerunan.

The constant of integration, c can be determined by substituting the given value of x with its corresponding value of y into the result of integration of the gradient function.

Contoh 8 :

Contoh 8

Tentukan nilai pemalar pengamiran, c bagi $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$ dengan $y = 25$ apabila $x = 2$.

Penyelesaian

Diberi $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$.

Jadi, $y = \int (4x^3 + 6x^2 - 3) dx$

$$y = \frac{4x^4}{4} + \frac{6x^3}{3} - 3x + c$$

$$y = x^4 + 2x^3 - 3x + c$$

Apabila $x = 2$ dan $y = 25$,

$$25 = 2^4 + 2(2)^3 - 3(2) + c$$

$$c = -1$$

Maka, nilai pemalar pengamiran, c

bagi $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$ ialah -1 .

Example 8 :

Example 8

Determine the constant of integration, c for $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$ where $y = 25$ when $x = 2$.

Solution

Given $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$.

Then, $y = \int (4x^3 + 6x^2 - 3) dx$

$$y = \frac{4x^4}{4} + \frac{6x^3}{3} - 3x + c$$

$$y = x^4 + 2x^3 - 3x + c$$

When $x = 2$ and $y = 25$,

$$25 = 2^4 + 2(2)^3 - 3(2) + c$$

$$c = -1$$

Thus, the constant of integration, c

for $\frac{dy}{dx} = 4x^3 + 6x^2 - 3$ is -1 .

Contoh 9 :

Contoh 9

Kecerunan bagi suatu lengkung pada titik (x, y) ialah $\frac{dy}{dx} = 15x^2 + 4x - 3$.

- (a) Jika lengkung itu melalui titik $(-1, 2)$, cari persamaan lengkung itu.
 (b) Seterusnya, cari nilai y apabila $x = 1$.

Penyelesaian

(a) Diberi $\frac{dy}{dx} = 15x^2 + 4x - 3$.

$$\text{Jadi, } y = \int (15x^2 + 4x - 3) dx$$

$$y = 5x^3 + 2x^2 - 3x + c$$

Apabila $x = -1$ dan $y = 2$,

$$2 = 5(-1)^3 + 2(-1)^2 - 3(-1) + c$$

$$c = 2$$

Maka, persamaan lengkung itu ialah

$$y = 5x^3 + 2x^2 - 3x + 2.$$

(b) Apabila $x = 1$,

$$y = 5(1)^3 + 2(1)^2 - 3(1) + 2$$

$$y = 6$$

Maka, $y = 6$ apabila $x = 1$.

Example 9 :

Example 9

The gradient function of a curve at point (x, y) is given by $\frac{dy}{dx} = 15x^2 + 4x - 3$.

- (a) If the curve passes through the point $(-1, 2)$, find the equation of the curve.
 (b) Subsequently, find the value of y when $x = 1$.

Solution

(a) Given $\frac{dy}{dx} = 15x^2 + 4x - 3$.

$$\text{Then, } y = \int (15x^2 + 4x - 3) dx$$

$$y = 5x^3 + 2x^2 - 3x + c$$

When $x = -1$ and $y = 2$,

$$2 = 5(-1)^3 + 2(-1)^2 - 3(-1) + c$$

$$c = 2$$

Thus, the equation of the curve is

$$y = 5x^3 + 2x^2 - 3x + 2.$$

(b) When $x = 1$,

$$y = 5(1)^3 + 2(1)^2 - 3(1) + 2$$

$$y = 6$$

Then, $y = 6$ when $x = 1$.

Latihan Kendiri 3.4 / Self-Exercise 3.4 :

1. Cari nilai pemalar pengamiran, c bagi fungsi kecerunan .

Find the constant of integration, c for the following gradient functions.

(a) $\frac{dy}{dx} = 4x - 2, y = 7$ apabila $x = -1$

$\frac{dy}{dx} = 4x - 2, y = 7$ when $x = -1$

(b) $\frac{dy}{dx} = -6x - \frac{6}{x^3}, y = 6$ when $x = -1$

$\frac{dy}{dx} = -6x - \frac{6}{x^3}, y = 6$ when $x = -1$

Jawapan/Answer: (a) 3 (b) 6

2. Diberi $\frac{dy}{dx} = 20x^3 - 6x^2 - 6$ dan $y = 2$ apabila $x = 1$, cari nilai y apabila $x = \frac{1}{2}$.

Given $\frac{dy}{dx} = 20x^3 - 6x^2 - 6$ and $y = 2$ when $x = 1$, find the value of y when $x = \frac{1}{2}$.

Jawapan/Answer: $\frac{33}{16}$

3. Cari persamaan lengkung bagi setiap fungsi kecerunan yang melalui titik berikut.

Find the equation of curve for each gradient function which passes through the given point.

(a) $\frac{dy}{dx} = 9x^2 - 2$, titik / at point (1, 6)

(b) $\frac{dy}{dx} = 10x - 2$, titik / at point (2, 13)

Jawapan/Answer: $y = 3x^3 - 2x + 5$

Jawapan/Answer: $y = 5x^2 - 2x - 3$

(c) $\frac{dy}{dx} = 24x^2 - 5$, titik / at point (1, 1)

(d) $\frac{dy}{dx} = 18x^2 + 10x$, titik / at point (-2, -10)

Jawapan/Answer: $y = 8x^3 - 5x - 2$

Jawapan/Answer: $y = 6x^3 + 5x^2 + 18$

Latihan Formatif 3.2 / Formative Exercise 3.2

1. Cari kamiran tak tentu bagi setiap yang berikut.

Find the indefinite integral for each of the following.

(a) $\int \frac{1}{2} dx$

(b) $\int \frac{5}{3x^3} dx$

(c) $\int \frac{1}{\sqrt{x}} dx$

(d) $\int \left(\frac{2}{x^3} - \frac{3}{x^4} \right) dx$

Jawapan/Answer: (a) $\frac{1}{2}x + c$ (b) $-\frac{5}{6x^2} + c$ (c) $2x^{\frac{1}{2}} + c$ (d) $-\frac{1}{x^2} + \frac{1}{x^3} + c$

2. Kamirkan setiap yang berikut terhadap x.

Integrate each of the following with respect to x.

(a) $\frac{5x^2 - 3x^3}{x}$

(b) $\frac{6x^3 + 2x^2}{2x^2}$

(c) $(5-6x)^3$

(d) $\frac{1}{\sqrt[4]{5-2x}}$

Jawapan/Answer: (a) $\frac{5}{2}x^2 - x^3 + c$ (b) $\frac{3}{2}x^2 + x + c$ (c) $-\frac{(5-6x)^4}{24} + c$ (d) $-\frac{2(5-2x)^{\frac{3}{4}}}{3} + c$

3. Diberi $\frac{dy}{dx} = 10x + \frac{p}{x^2}$, dengan keadaan p ialah pemalar. Jika $\frac{dy}{dx} = 20\frac{1}{2}$ dan $y = 19$ apabila $x = 2$, cari nilai p .
Seterusnya, cari nilai y apabila $x = -2$.

It is given that $\frac{dy}{dx} = 10x + \frac{p}{x^2}$, where p is a constant. If $\frac{dy}{dx} = 20\frac{1}{2}$ and $y = 19$ when $x = 2$, find the value of p .
Subsequently, find the value of y when $x = -2$.

Jawapan/Answer: $p = 2, y = 21$

4. (a) Diberi $\frac{dy}{dx} = 4x^3 - 15x^2 + 6$ dan $y = -20$ apabila $x = 3$, cari nilai y apabila $x = -2$.
Given $\frac{dy}{dx} = 4x^3 - 15x^2 + 6$ and $y = -20$ when $x = 3$, find the value of y when $x = -2$.

(b) Diberi $\frac{dy}{dx} = 2x + 2$ dan $y = 2$ apabila $x = 2$, cari nilai x apabila $y = -6$.
Given $\frac{dy}{dx} = 2x + 2$ and $y = 2$ when $x = 2$, find the value of x when $y = -6$.

Jawapan/Answer: 60

Jawapan/Answer: $x = 0, -2$

5. Rajah di sebelah menunjukkan suatu lengkung yang melalui titik A(1, -1).

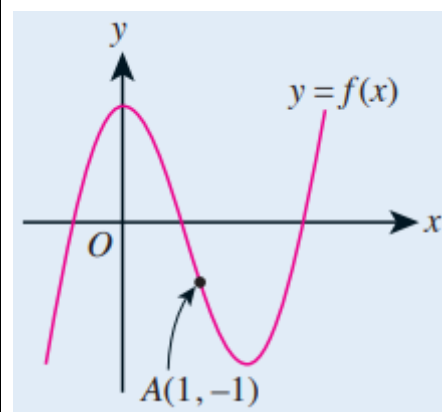
Diberi fungsi kecerunan bagi lengkung tersebut ialah $\frac{dy}{dx} = 3x^2 - 8x$, cari

persamaan bagi lengkung tersebut.

The diagram on the right shows a curve that passes through point A(1, -1). Given

the gradient function of that curve is $\frac{dy}{dx} = 3x^2 - 8x$, find the equation for that

curve.



Jawapan/Answer: $y = x^3 - 4x^2 + 2$

6. Diberi kecerunan normal bagi suatu lengkung pada suatu titik ialah $\frac{1}{6x-2}$. Jika lengkung itu melalui titik (2, 2), cari persamaan bagi lengkung tersebut.

It is given that the gradient of a normal to a curve at one point is $\frac{1}{6x-2}$. If the curve passes through point (2, 2), find the equation for that curve.

Jawapan/Answer: $y = 2x - 3x^2 + 10$

7. Diberi fungsi kecerunan bagi suatu lengkung ialah $ax + b$. Kecerunan lengkung pada titik (-2, 8) ialah -7 dan kecerunan lengkung pada titik (0, 6) ialah 5. Cari nilai a dan nilai b. Seterusnya, cari persamaan bagi lengkung tersebut.

It is given that the gradient function of a curve is $ax + b$. The gradient of the curve at (-2, 8) is -7 and the gradient of the curve at (0, 6) is 5. Find the values of a and b. Then, find the equation for that curve.

Jawapan/Answer: $a = 6, b = 5, y = 3x^2 + 5x + 6$

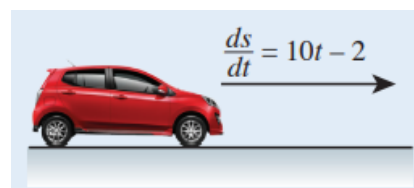
8. Rajah di sebelah menunjukkan sebuah kereta yang dipandu di sebuah jalan raya yang lurus. Diberi fungsi perubahan sesaran bagi kereta tersebut ialah

$$\frac{ds}{dt} = 10t - 2 \text{ dan } s = 8\text{m when } t = 1\text{s. Cari sesaran, dalam m, apabila } t = 3\text{ s.}$$

The diagram on the right shows a car being driven on a straight road. It is given that the

rate of change of the displacement function of the car is $\frac{ds}{dt} = 10t - 2$ and $s = 8\text{ m}$

when $t = 1\text{ s}$. Find the displacement, in m, when $t = 3\text{ s}$.



Jawapan/Answer : 44m

SOALAN BERFORMAT SPM

SPM FORMAT QUESTIONS

Paper 1

1. Tunjukkan bahawa $\frac{d}{dx} \left(\frac{x^2}{3+2x} \right) = \frac{2x^2+6x}{(3+2x)^2}$. Seterusnya, cari $\int \frac{x(x+3)}{(3+2x)^2} dx$.

Show that $\frac{d}{dx} \left(\frac{x^2}{3+2x} \right) = \frac{2x^2+6x}{(3+2x)^2}$. Hence, find $\int \frac{x(x+3)}{(3+2x)^2} dx$.

[4 markah / marks]

Jawapan/Answer : $\frac{1}{2} \left(\frac{x^2}{3+2x} \right)$

2. Cari kamiran tak tentu bagi setiap yang berikut :

Find the indefinite integral for each of the following :

(a) $\int \left(\frac{2}{3}x^5 + \frac{1}{6x^5} \right) dx$ (b) $\int \left(\frac{(x^2+3)^2}{x^2} \right) dx$ (c) $\int \frac{3}{2} \left(5 - \frac{2}{3}u \right)^5 du$ [6 markah /

marks]

Jawapan/Answer : (a) $\frac{x^6}{9} - \frac{1}{24x^4} + c$ (b) $\frac{x^3}{3} + 6x - \frac{9}{x} + c$ (c) $-5 \left(5 - \frac{2}{3}u \right)^6 + c$

3. Diberi $\int (5x^3 + 2)dx = kx^4 + 2x + c$, dengan keadaan k dan c ialah pemalar. Cari

Given that $\int (5x^3 + 2)dx = kx^4 + 2x + c$, where k and c are constants. Find

(a) nilai k ,
the value of k ,

(b) nilai c jika $\int (5x^3 + 2)dx = 30$ apabila $x = 2$.
the value of c if $\int (5x^3 + 2)dx = 30$ when $x=2$.

[3 markah / marks]

Jawapan/Answer: (a) $k = \frac{5}{4}$ (b) $c = 6$

4. Jika $\frac{dy}{dx} = 4x^3 - 4x$ dan $y = 0$ apabila $x = 2$, cari y dalam sebutan x .

If $\frac{dy}{dx} = 4x^3 - 4x$ and $y = 0$ when $x = 2$, find y in terms of x .

[3 markah / marks]

Jawapan/Answer: $y = x^4 - 2x^2 - 8$

5. Jika $\frac{dp}{dv} = 2v - \frac{v^3}{2}$ dan $p = 0$ apabila $v = 0$, cari nilai p apabila $v = 1$.

If $\frac{dp}{dv} = 2v - \frac{v^3}{2}$ and $p = 0$ when $v = 0$, find the value p when $v = 1$.

[3 markah / marks]

Jawapan/Answer: $p = \frac{7}{8}$

6. Cari persamaan bagi lengkung yang mempunyai fungsi kecerunan $2x^2 + 3x - 1$ dan melalui titik asalan.

Find the equation of the curve with gradient function $2x^2 + 3x - 1$, which passes through the origin. [3 markah / marks]

Jawapan/Answer: $y = \frac{2}{3}x^3 + \frac{3}{2}x^2 - x$

7. Diberi $\frac{d^2y}{dx^2} = 4x$, dan $\frac{dy}{dx} = 0$, $y = 2$ apabila $x = 0$. Cari $\frac{dy}{dx}$ dan y dalam sebutan x .

Given that $\frac{d^2y}{dx^2} = 4x$, and that $\frac{dy}{dx} = 0$, $y = 2$ when $x = 0$. Find $\frac{dy}{dx}$ and y in terms of x . [4 markah / marks]

Jawapan/Answer: $y = \frac{2}{3}x^3 + 2$

Soalan SPM 2003 K1 No 17

Diberi $\int \left(\frac{5}{(1+x)^4} \right) dx = k(1+x)^n + c$, dengan keadaan k dan n ialah pemalar. Cari nilai k dan nilai n .

Given that $\int (5x^3 + 2) dx = kx^4 + 2x + c$, where k and c are constants. Find the value of k and n .

[3 markah / marks]

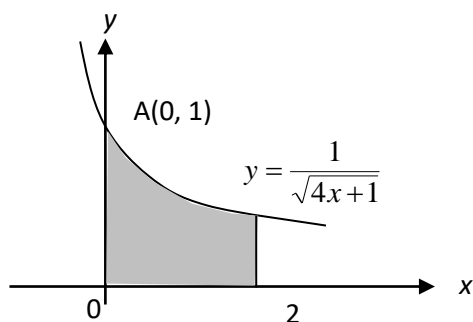
Jawapan/Answer: $k = -\frac{5}{3}, n = -3$

Paper 2

1. Suatu lengkung dengan keadaan $\frac{dy}{dx} = 1 + \frac{1}{2x^2}$. Diberi lengkung tersebut melalui titik $(-1, \frac{1}{2})$, cari persamaan bagi lengkung tersebut.

A curve is such that $\frac{dy}{dx} = 1 + \frac{1}{2x^2}$. Given that the curve passes through the point $(-1, \frac{1}{2})$, find the equation of the curve.

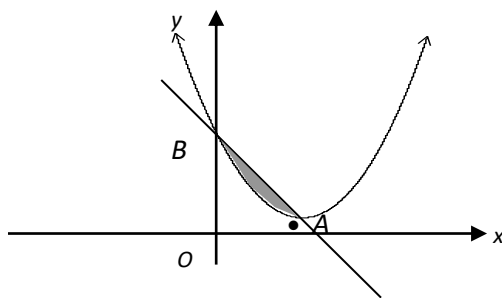
[4 markah / marks]



Jawapan/Answer: $y = x - \frac{1}{2x} + 1$

2. Rajah di bawah menunjukkan suatu lengkung dengan keadaan $\frac{dy}{dx} = 2x - 6$. Titik minimum lengkung tersebut ialah A(3, 1). AB ialah suatu garis lurus yang melalui titik A dan B, dengan keadaan B ialah titik persilangan antara lengkung tersebut dengan paksi-y.

The diagram shows a curve such that $\frac{dy}{dx} = 2x - 6$. The minimum point of the curve is A(3,1). AB is a straight line passing through A and B, where B is the point of intersection between the curve and y-axis.



Cari persamaan bagi lengkung tersebut.
Find the equation of the curve.

[3 markah / marks]

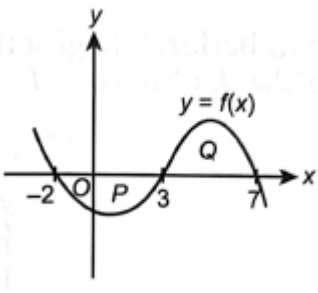
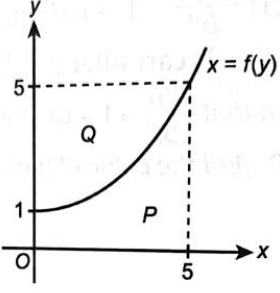
Jawapan/Answer: $y = x^2 - 6x + 10$

<p>(c) $\int_0^2 \pi(x^2 - 2x)^2 dx$ [$1\frac{1}{15}\pi$]</p>	<p>(d) $\int_{-2}^1 \frac{x^4 - x}{x} dx$ [$-6\frac{3}{4}$]</p>
<p>(e) $\int_1^2 \frac{1}{(3x-4)^2} dx$ [-0.5]</p>	<p>(f) $\int_0^1 \frac{10}{(2-5x)^3} dx$ [$-\frac{5}{36}$]</p>
<p>(g) $\int_1^4 (\sqrt{x} - 2) dx$ [$-\frac{4}{3}$]</p>	<p>(h) $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$ [2]</p>
<p>Cuba Latihan Kendiri 3.5 ms 95</p>	

2. Diberi $\int_{-1}^4 f(x) dx = 6$ dan $\int_0^3 g(x) dx = 2$, cari nilai bagi setiap yang berikut :
Given that $\int_{-1}^4 f(x) dx = 6$ and $\int_0^3 g(x) dx = 2$, find the following values :

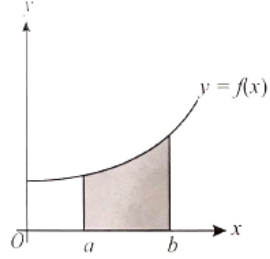
<p>Contoh Example</p> $\int_{-1}^4 2 f(x) dx$ $= 2 \int_{-1}^4 f(x) dx$ $= 2 (6)$ $= 12$	<p>(a) $\int_{-1}^1 f(x) dx + \int_1^4 f(x) dx$ [6]</p>	<p>(b) $\int_4^{-1} f(x) dx + \int_0^3 g(x) dx$ [-4]</p>
<p>(c) $\int_0^3 [\frac{1}{2}g(x) - x^2] dx$ [-8]</p>	<p>(d) $\int_{-1}^4 [1 - 2f(x)] dx$ [-7]</p>	<p>(e) $\int_0^3 g(x) dx \times \int_{-1}^4 f(x) dx$ [12]</p>

<p>3. Selesaikan yang berikut : <i>Solve the following :</i></p>	
<p>(a) Diberi bahawa $\int_2^7 f(x) dx = 10$, cari <i>Given that $\int_2^7 f(x) dx = 10$, find</i></p> <p>i. Nilai $\int_7^2 f(x) dx$ [-10] <i>The value of $\int_7^2 f(x) dx$</i></p> <p>ii. Nilai k jika $\int_2^7 [f(x) - k] dx = 25$ [-3] <i>The value of k if $\int_2^7 [f(x) - k] dx = 25$</i></p>	<p>(b) Diberi bahawa $\int_1^3 f(x) dx = 4$, cari <i>Given that $\int_1^3 f(x) dx = 4$, find</i></p> <p>i. Nilai $\int_1^3 2f(x) dx$ [8] <i>The value of $\int_1^3 2f(x) dx$</i></p> <p>ii. Nilai $\int_1^3 [1 + f(x)] dx$ [6] <i>The value of $\int_1^3 [1 + f(x)] dx$</i></p>

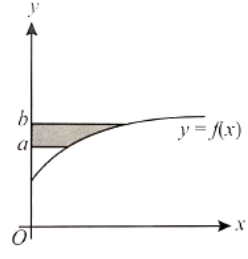
<p>(c)</p>  <p style="text-align: right;">[8, -4.5, 3.5]</p> <p>Diberi luas P ialah 4.5 unit² dan luas Q ialah 8 unit². Cari kamiran tentu <i>Given that the area of P is 4.5 unit² and area of of Q is 8 unit². Find the definite integral</i></p> <p>(i) $\int_3^7 f(x) dx$ (ii) $\int_{-2}^3 f(x) dx$ (iii) $\int_{-2}^7 f(x) dx$</p>	<p>(d)</p>  <p style="text-align: right;">[7, Luas Q = $\int_1^5 x dy$]</p> <p>Diberi luas Q ialah 18 unit², <i>Given that the area Q is 18 unit²,</i></p> <p>(i) Cari luas P <i>Find the area of P</i> (ii) ungkapkan luas Q dalam tatatanda kamiran <i>Express the area of Q in integration notation</i></p>
<p>(e) Diberi $\frac{d}{dx} \left(\frac{x}{x+1} \right) = f(x)$, hitungkan nilai $\int_1^2 f(x) dx$ [$\frac{1}{6}$] <i>Given that $\frac{d}{dx} \left(\frac{x}{x+1} \right) = f(x)$, calculate the value of $\int_1^2 f(x) dx$</i></p>	<p>(f) Diberi $\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{4}f(x)$, hitungkan nilai $\int_2^3 f(x) dx$ [$\frac{1}{2}$] <i>Given that $\frac{d}{dx} \left(\frac{x}{x+1} \right) = \frac{1}{4}f(x)$, calculate the value of $\int_2^3 f(x) dx$</i></p>

4. Cari luas rantau berlorek
Find the area of shaded region

Nota Imbasan Smart Note

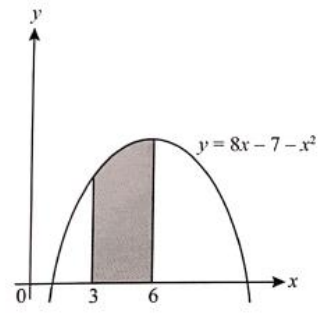


Luas di antara lengkung dengan paksi x
Area between the curve and the x axis
 $L = \int_a^b y \, dx$



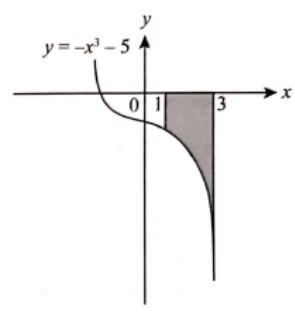
Luas di antara lengkung dengan paksi y
Area between the curve and the y axis
 $L = \int_a^b x \, dy$

Contoh Example



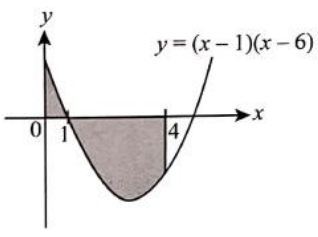
Luas Area
 $= \int_3^6 (8x - 7 - x^2) \, dx$
 $= [4x^2 - 7x - \frac{x^3}{3}]_3^6$
 $= \{ [4(6)^2 - 7(6) - \frac{6^3}{3}] - [4(3)^2 - 7(3) - \frac{3^3}{3}] \}$
 $= 24 \text{ unit}^2$

(a)



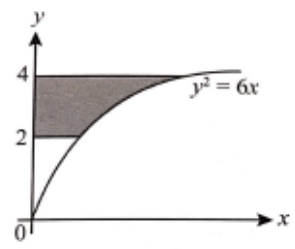
[30]

(b)

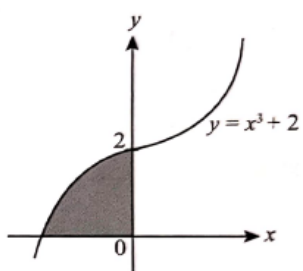
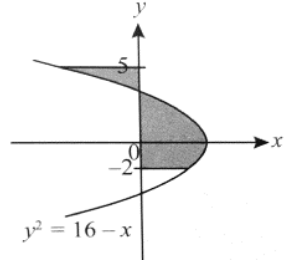


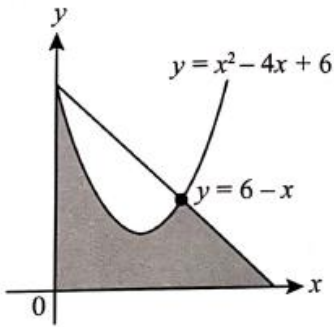
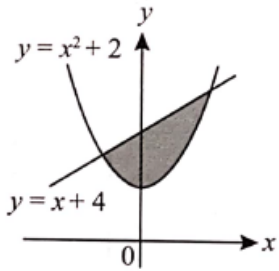
[16 $\frac{1}{3}$]

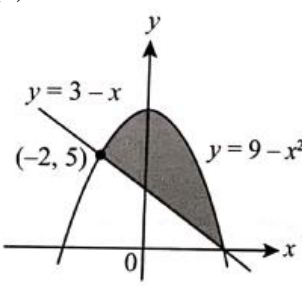
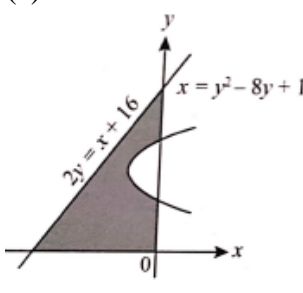
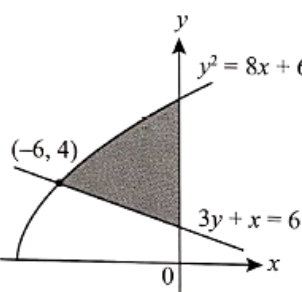
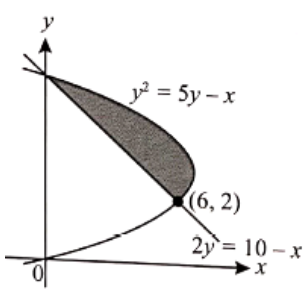
(c)



[3 $\frac{1}{9}$]

<p>(d)</p>  <p style="text-align: right;">[1.89]</p>	<p>(e)</p>  <p style="text-align: right;">[76 $\frac{1}{3}$]</p>
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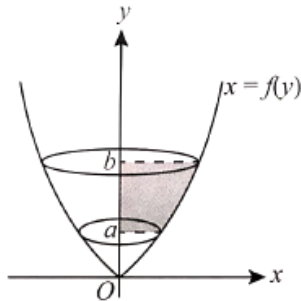
<p>5. Cari luas rantau berlorek yang dibatasi oleh lengkung dan garis lurus <i>Find the area of shaded region that bounded by a curve and a straight line.</i></p>	
<p>Contoh Example</p>	
 <p>Titik Persilangan <i>Point of intersection</i></p> $x^2 - 4x + 6 = 6 - x$ $x^2 - 3x = 0$ $x(x - 3) = 0$ <p>$x = 0$ atau $x = 3$</p> <p>(0,6) atau (3,3)</p> <p>Luas di bawah lengkung + luas segitiga <i>Area under curve + area of triangle</i></p> $= \int_0^3 (x^2 - 4x + 6) dx + \frac{1}{2} (3)(3)$ $= \left[\frac{x^3}{3} - \frac{4x^2}{2} + 6x \right]_0^3 + 4.5$ $= \left\{ \left[\frac{3^3}{3} - 2(3)^2 + 6(3) \right] - \left[\frac{0^3}{3} - 2(0)^2 + 6(0) \right] \right\} + 4.5$ $= 13.5 \text{ unit}^2$	<p>(a)</p>  <p style="text-align: right;">[4.5]</p>

<p>(b)</p>  <p>$y = 3 - x$ $(-2, 5)$ $y = 9 - x^2$</p> <p style="text-align: center;">$[20\frac{5}{6}]$</p>	<p>(c)</p>  <p>$x = y^2 - 8y + 12$ $2y = x + 16$</p> <p style="text-align: center;">$[53\frac{1}{3}]$</p>
<p>(d)</p>  <p>$y^2 = 8x + 64$ $(-6, 4)$ $3y + x = 6$</p> <p style="text-align: center;">$[7\frac{1}{3}]$</p>	<p>(e)</p>  <p>$y^2 = 5y - x$ $(6, 2)$ $2y = 10 - x$</p> <p style="text-align: center;">$[4.5]$</p>

6. Cari isipadu janaan apabila luas rantau di bawah lengkung diputarkan melalui 360° pada paksi x atau paksi y.

Find the generated volume when the area under curve is rotated through 360° about the x axis or y axis.

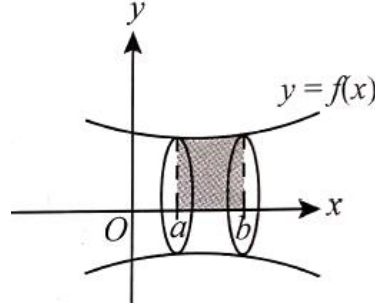
Nota Imbasan Smart Note



Isi padu janaan apabila luas di bawah lengkung diputarkan pada paksi y

Volume of revolution when area under curve is revolved about the y axis

$$V = \int_a^b \pi x^2 dy$$

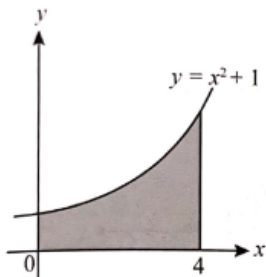


Isi padu janaan apabila luas di bawah lengkung diputarkan pada paksi x

Volume of revolution when area under curve is revolved about the x axis

$$V = \int_a^b \pi y^2 dx$$

Contoh Example



Isipadu Volume

$$= \int_0^4 \pi(x^2 + 1)^2 dx$$

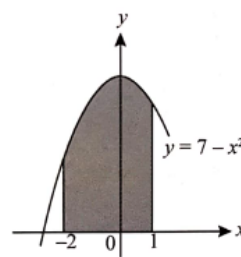
$$= \int_0^4 \pi(x^4 + 2x^2 + 1) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x \right]_0^4$$

$$= \pi \left\{ \left[\frac{4^5}{5} + \frac{2(4)^3}{3} + 4 \right] - \left[\frac{0^5}{5} + \frac{2(0)}{3} + 0 \right] \right\}$$

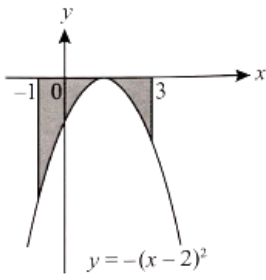
$$= 251 \frac{7}{15} \text{ unit}^3$$

(a)



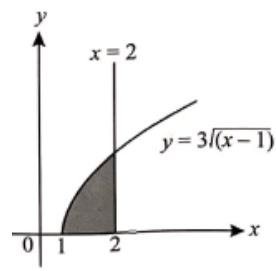
$$\left[111 \frac{3}{5} \pi \right]$$

(b)



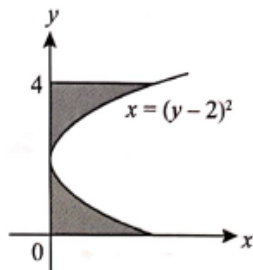
$[48\frac{4}{5}\pi]$

(c)



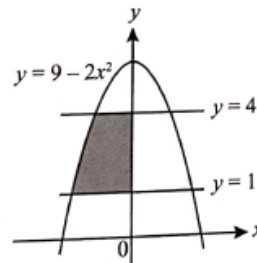
$[4.5\pi]$

(d)



$[6.4\pi]$

(e)

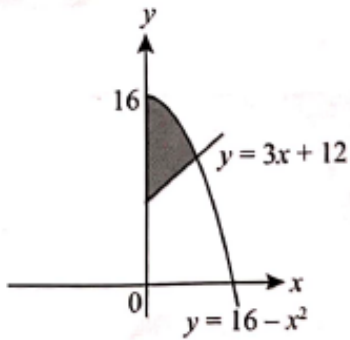


$[9.75\pi]$

7. Tentukan isipadu janaan bagi setiap yang berikut.

Determine the generated volume for each of the following.

Contoh *Example*



$$3x + 12 = 16 - x^2$$

$$x^2 + 3x - 4 = 0$$

$$(x - 1)(x + 4) = 0$$

$$x = 1 \text{ atau } x = 4$$

$$y = 15 \qquad y = 0$$

Isipadu janaan / *Generated volume*

= isipadu bagi luas di bawah lengkung + isipadu kon

volume of area under curve + volume of kon

$$= \int_{15}^{16} \pi x^2 dy + \frac{1}{3} \pi j^2 h$$

$$= \int_{15}^{16} \pi (16 - y)^2 dy + \frac{1}{3} \pi (1)^2 (3)$$

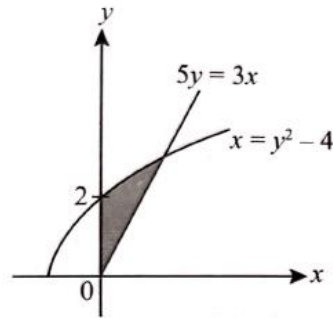
$$= \pi \left[16y - \frac{y^2}{2} \right]_{15}^{16} + \pi$$

$$= \pi \left\{ \left[16(16) - \frac{16^2}{2} \right] - \left[16(15) - \frac{15^2}{2} \right] \right\} + \pi$$

$$= 0.5 \pi + \pi$$

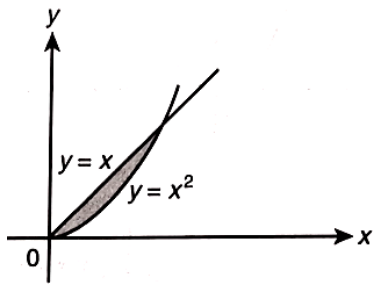
$$= 1.5 \pi$$

(a)



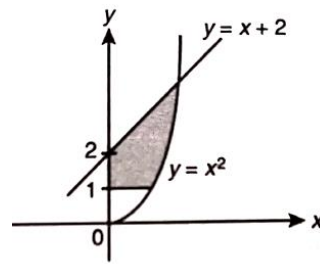
$$\left[17\frac{7}{15}\pi \right]$$

(b)



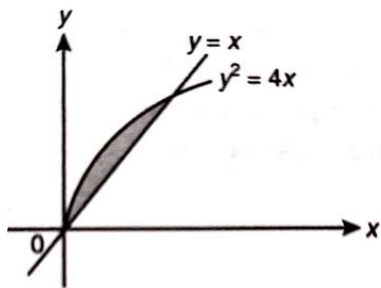
$$\left[\frac{2}{15}\pi \right]$$

(c)



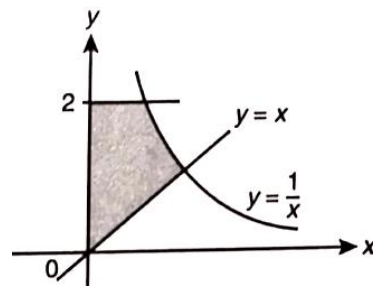
$$\left[4\frac{5}{6}\pi \right]$$

(d)



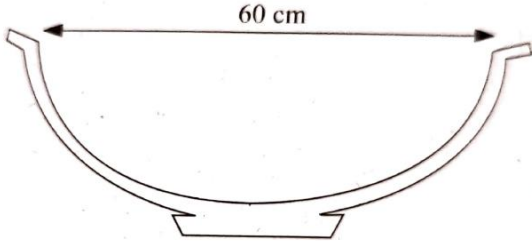
$$\left[8\frac{8}{15}\pi \right]$$

(e)

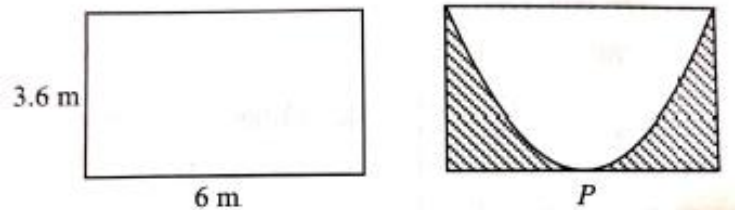


$$\left[\frac{5}{6}\pi \right]$$

3.4 Aplikasi Pengamiran <i>Application of integration</i>
Nota Imbasan <i>Smart Note</i>
Contoh <i>Example</i>
<ul style="list-style-type: none"> ❖ Gerakan linear. <i>Linear motion</i> ❖ Industri pembuatan. <i>Manufacturing industry</i> ❖ Bidang pembinaan. <i>Field of construction</i>
1. Selesaikan masalah berikut yang melibatkan pengamiran : <i>Solve the following problems involving integration</i>

(a)	<p>Kadar peningkatan populasi, $\frac{dP}{dt}$, di mana P ialah bilangan penduduk di suatu tempat dan t ialah masa. Diberi $\frac{dP}{dt} = \frac{1}{(t+1)^2}$ dan P = 9999 apabila t = 0, tunjukkan bahawa $P = 10\,000 - \frac{1}{t+1}$.</p>
(b)	<div style="text-align: center;">  </div> <p>Rajah menunjukkan pandangan sisi permukaan bagi sebuah mangkuk yang boleh diwakili oleh persamaan $y = ax^2$. Kelebaran dan kedalaman mangkuk ialah 60 cm dan 30 cm.</p> <ol style="list-style-type: none"> i. Tunjukkan bahawa $a = \frac{1}{30}$ ii. Tentukan isi padu, cm^3 air yang diperlukan bagi mengisi mangkuk itu dengan kedalaman 20cm. <p style="text-align: right;">$[6000\pi \text{ cm}^3]$</p> <p><i>Diagram shows a side elevation of the inner surface of a bowl which can be represented by the equation $y = ax^2$. The width and the depth of the bowl is 60cm and 30cm.</i></p> <ol style="list-style-type: none"> i) Show that $a = \frac{1}{30}$ ii) Determine the volume, in cm^3, of water needed to fill the bowl to a depth of 20 cm.

(c)



Rajah menunjukkan sebuah dinding bangunan dengan panjang 6m dan tinggi 3.6m. Shafiq diupah untuk mengecat dinding itu mengikut bentuk yang ditunjukkan dalam rajah P. Diberi lengkung itu berbentuk parabola dengan P ialah titik minimum.

- i) Cari luas, dalam m^2 , rantau berlorek yang akan dicat oleh Shafiq.
- ii) Sekiranya bayaran sebanyak RM170 dibayar untuk mengecat kawasan seluas $10m^2$, berapakah bayaran yang akan diperolehi oleh Shafiq.

[7.2 m^2 , RM122.40]

Diagram shows the wall of a building which is 6m long and 3.6m tall. Shafiq is hired to paint the wall by following the shape in the diagram P. Given the curve is parabolic where P is the minimum point.

- i) Find the area, in m^2 , of the shaded region where Shafiq will paint.
- ii) If a payment of RM170 is paid for painting an area of $10m^2$, how much the payment that Shafiq will received ?